Philosophical Foundations of Loop Quantum Gravity

圈量子引力的哲学基础

Carlo Rovelli and Francesca Vidotto

卡洛·罗韦利、弗兰切斯卡·维达托

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The University of Western Ontario is located in the traditional lands of Anishinaabek, Haudenosaunee, Lūnaapéewak and Attawandaron peoples.

西安大略大学坐落于阿尼什纳贝克、豪德诺索尼、伦纳佩瓦和阿塔万德隆民族的传统领地之上。

C. Rovelli (∅)

C. 罗韦利 (∅)

Aix-Marseille University, Marseille, France

法国马赛艾克斯-马赛大学

Université de Toulon, CPT-CNRS, La Valette-du-Var, France

法国拉瓦莱特迪瓦尔，土伦大学 CPT-CNRS 实验室

The Perimeter Institute, Waterloo, ON, Canada

加拿大安大略省滑铁卢，圆周理论物理研究所

Department of Philosophy, Rotman Institute of Philosophy, The University of Western Ontario, London, ON, Canada

加拿大安大略省伦敦市，韦仕敦大学罗特曼哲学研究所哲学系

e-mail: crovelli@uwo.ca

电子邮箱:crovelli@uwo.ca

F. Vidotto

F. 维达托

Department of Physics and Astronomy, Department of Philosophy, Rotman Institute of

韦仕敦大学物理与天文学系、罗特曼哲学系

Philosophy, The University of Western Ontario, London, ON, Canada

加拿大安大略省伦敦市，韦仕敦大学哲学系

e-mail: fvidotto@uwo.ca

电子邮箱:fvidotto@uwo.ca

Abstract

摘要

Understanding the quantum aspects of gravity is not only a matter of equations and experiments. Gravity is intimately connected with the structure of space and time, and understanding quantum gravity requires us to find a conceptual structure appropriate to make sense of the quantum aspects of space and time. In the course of the last decades, an extensive discussion on this problem has led to a clear conceptual picture that provides a coherent conceptual foundation of today's loop quantum gravity. We review this foundation, addressing issues such as the sense in which space and time are emergent, the notion of locality, the role of truncation that enables physical computations on finite graphs, the problem of time, and the characterization of the observable quantities in quantum gravity.

理解引力的量子属性不仅是方程与实验的问题。引力与时空结构密不可分，理解量子引力要求我们找到一套恰当的概念结构，以厘清空间和时间量子属性的意义。过去数十年来，针对该问题的大量讨论已经形成了清晰的概念图景，为当代圈量子引力提供了连贯的概念基础。本文综述这一基础，探讨以下问题: 空间和时间以何种方式涌现、局域性的概念、可在有限图上进行物理计算的截断方法、时间问题，以及量子引力中可观测量的特征。

Keywords

关键词

Quantum gravity · Time · Observables · Relations

量子引力 · 时间 · 可观测量 · 关系

Introduction

引言

What is a quantum spacetime? Are space and time emergent notions? If so, what do they emerge from, and in which sense? Is there a fundamental ontology, from which conventional space and time emerge? Does a quantum theory of gravity require a specific time variable, as the Schr'odinger equation does? If not, what is the connection between the common notion of time and quantum gravity? How is evolution described, in the absence of a canonical time or a fixed background spacetime structure? Which empirically observable quantities are well defined in a quantum spacetime? How do we compute their behavior? Are they local in some sense?

什么是量子时空？空间和时间是涌现性概念吗？如果答案是肯定的，那它们从何涌现，又在何种意义上是涌现的？是否存在某种基础本体，常规的空间和时间由它涌现而来？正如薛定谔方程那样，量子引力理论是否要求一个特定的时间变量？如果不需要，常用的时间概念与量子引力之间存在怎样的关联？在没有正则时间或固定背景时空结构的情况下，如何描述演化？哪些经验可观测物理量在量子时空中是良定义的？我们该如何计算它们的行为？这些量在某种意义上是定域的吗？

These questions have been discussed widely and at length in the quantum gravity literature and routinely confuse those entering the field anew. Here we address them, showing how a coherent conceptual framework for a quantum theory of gravity can be cleanly defined.

这些问题已经在量子引力文献中得到了广泛充分的讨论，却一直困扰着刚进入该领域的研究者。本文将探讨这些问题，说明如何清晰定义量子引力的自洽概念框架。

We give a basic discussion of the notions of "space" and "time." This is essential because substantial confusion derives from mixing up different ways in which these notions are used. We discuss observability in general relativistic physics and in quantum mechanics. Observability in quantum gravity is subtle precisely because it combines the conceptual subtleties of general relativity with those in quantum mechanics. We discuss the notion of "emergence" and the so-called problem of time, the precise role of the finite combinatorial structures (graphs and two complexes) that enter concrete calculations, their relation to locality, and the reason they are physically relevant. Our focus is on the conceptual structure of quantum gravity in the loop formulation (LQG) [1-4].

我们对“空间”和“时间”的概念做基础讨论——这一讨论十分必要，因为混淆这两个概念的不同用法已经引发了大量困惑。我们讨论广义相对论物理学和量子力学中的可观测性。量子引力中的可观测性之所以微妙，正是因为它结合了广义相对论和量子力学二者的概念难点。我们讨论“涌现”概念和所谓的时间问题，探讨具体计算中用到的有限组合结构 (图和二维复形) 的准确作用、它们与定域性的关联，以及它们具备物理相关性的原因。我们的研究重点是圈量子引力 (LQG) 框架中量子引力的概念结构 [1-4]。

Two Distinct Notions of Space

两种不同的空间概念

In its simplest usage, space is the structure determined by a relation of contiguity between physical entities. We use this notion when we say ”I am in London,” or ’the electron has reached the detector.” In these cases we spatially locate entities (our self, London, an electron, a detector) with respect to one another. This notion of space does not involve metric quantities (distances, areas, volumes, etc.) and refers to a relation between entities. This is a relational notion of space.

最简单的用法中，空间是由物理实体之间的邻接关系所确定的结构。当我们说“我在伦敦”，或者“电子已经到达探测器”时，我们使用的就是这个概念。在这些例子中，我们是在实体 (我们自身、伦敦、电子、探测器) 之间相互标定空间位置。这个空间概念不涉及度量量 (距离、面积、体积等)，仅指代实体之间的关系，这是一种关系论空间概念。

In Newtonian physics, a different notion of space is employed. ”Space,” in this more specific usage, is a container in which things, or observations, are located; it is an entity assumed to exists by itself, independently from the objects or the dynamical degrees of freedom. In this sense, ”space” can also be empty. Dynamical objects, or observations, are located ”in space,” namely, they are located with respect to it. Space, in this sense, can have a geometrical structure, which in Newtonian physics is described by three-dimensional Euclidean geometry. This notion of container space, formalized by Newton (anticipated by ancient atomism), has played a fundamental role in the development of physics. We employ it for instance when we use the Newton equations to describe the dynamical of particles as (the evolution in time of) their location in $\mathbb{R}^3$ (In the philosophical literature there are other, distinct, discussions regarding relational aspects of space. One is about the relational aspects of geometry. Another discussion is the confusion between the velocity being relative and acceleration being relative. A third one regards the possibility of a relational reading of Newtonian mechanics (in terms of reference systems). Here we are not referring to these issues. The relation we refer to is simple contiguity, which does not require any metric connotation. From the perspective of quantum gravity, Newtonian space is better understood as special configuration of the gravitational field: an entity. All these discussions can be traced to the famous distinction at the beginning of the Principia [5] and to Leibniz’s relationalism: ”As for my own opinion, I have said more than once, that I hold space to be something merely relative, as time is, that I hold it to be an order of coexistences, as time is an order of successions.” Third Paper, paragraph 4; G VII.363 pg 25-26 [6]. Mixing up these various discussions is an endless source of confusion.).

牛顿物理学中采用了另一种不同的空间概念。在这个更具体的用法中，“空间”是容纳事物或观测的容器；它是一个独立存在的实体，不依赖于物体或动力学自由度。在这个意义上，“空间”也可以是空的。动力学物体或观测都位于“空间之中”，也就是它们相对于空间来定位。这种意义上的空间可以具有几何结构，在牛顿物理学中由三维欧几里得几何描述。这种由牛顿 (早自古原子论就已有雏形) 系统化的容器空间概念，在物理学发展中发挥了基础性作用。例如当我们使用牛顿方程描述粒子在 $\mathbb{R}^3$ 中位置的动力学 (位置随时间的演化) 时，我们用的就是这个概念。(哲学文献中还有其他关于空间关系性的不同讨论: 一是关于几何的关系性；二是关于速度相对性与加速度相对性的混淆；三是关于牛顿力学能否以关系论从参考系角度解读。本文不讨论这些问题，本文所说的关系仅指简单的邻接关系，不要求任何度量内涵。从量子引力的视角看，牛顿空间更好地被理解为引力场的特殊组态，本身就是一个实体。所有这些讨论都可以追溯到《原理》[5] 开篇著名的区分，以及莱布尼茨的关系论: “至于我自己的观点，我已经不止一次说过，我认为空间和时间一样仅仅是相对的: 空间是并存事物的秩序，时间是接续事件的秩序。”引自《第三篇论文》，第 4 段；G VII.363，第 25-26 页 [6]。混淆这些不同讨论只会带来无尽的困惑。)

The distinction between these two notions of space (relational and Newtonian) is essential to get clarity in quantum gravity. In brief: the first extends to the full quantum gravity regime, the second does not [7]. The reason the relational notion of space survives in quantum gravity is that contiguity - and therefore localization - can be defined with respect to Newtonian or classical general relativistic spacetime, but can also be defined with respect to other entities, even in the absence of such Newtonian or classical general relativistic spacetime. On the other hand, the reason the Newtonian ”container” notion of space needs to be abandoned is that Newtonian space is recognized on physical grounds to be an approximate description of a particular configuration of a quantum field.

这两种空间概念 (关系论空间与牛顿空间) 的区分，对厘清量子引力研究至关重要。简言之: 第一种概念可以推广到完整的量子引力区域，第二种则不能 [7]。关系论空间能在量子引力中保留的原因是: 邻接性以及由此产生的定位，既可以相对于牛顿或经典广义相对论时空定义，也可以在不存在这类时空的情况下，相对于其他实体定义。另一方面，牛顿“容器”空间概念需要被抛弃的原因是: 从物理层面看，牛顿空间只是对量子场某一特定组态的近似描述。

In its Newtonian version, the second of these notions of space emerges, from the fundamental theory within a series of approximations:

牛顿版本的第二种空间概念，是基础理论经过一系列近似后涌现出来的:

1. Newtonian space emerges from Minkowski space in the low relative-velocity approximation (formally, this is a $c \to \infty$ limit, where c is the speed of light).

1. 牛顿空间是闵氏空间在相对低速近似下涌现的结果 (形式上这是一个 $c \to \infty$ 极限，其中 c 是光速)。

2. Minkowski space emerges from a general relativistic (pseudo-)Riemannian geometry at small scales with respect to the curvature radius (formally, it can be identified with the tangent space at a point).

2. 闵氏空间是广义相对论的 (伪) 黎曼几何在远小于曲率半径的尺度上涌现的结果 (形式上，它等同于某一点的切空间)。

3. A (pseudo-)Riemannian geometry emerges from the quantum geometry defined by the LQG states and their dynamics, in a suitable classical limit (formally, this is a $\hbar \to 0$ limit).

3. (伪) 黎曼几何是 LQG 量子几何 (由 LQG 态及其动力学定义) 在合适的经典极限下涌现的结果 (形式上，这是一个 $\hbar \to 0$ 极限)。

The relevant notion of ”emergence” here is a weak one, common in physics: in some contexts, but not always, the physical system admits a convenient and effective approximate description in terms of an ”emergent” theory. The emergent theory (here general relativity) is self-standing and autonomous and utilizes its own proper notions (here relativistic spacetime). These can be related to notions of the underpinning theory: they approximate certain (not all) particular configurations of those.

这里所说的“涌现”是物理学中常见的弱涌现: 在某些 (而非所有) 情境下，物理系统可以用“涌现”理论给出方便有效的近似描述。涌现理论在这里是广义相对论，是自足自主的，使用自身特有的概念，在这里是相对论时空。这些概念可以关联到基础理论的概念: 它们是基础理论特定 (而非全部) 组态的近似。

Notice that the Newtonian intuition of the existence of space as an entity is not contradicted neither by general relativity nor by LQG. Newtonian space is simply better understood in these theories as the approximate description of a - classical, or, respectively, quantum - dynamical field. With respect to this field, location can be defined relationally, as it can be defined relationally with respect to anything else.

请注意，无论是广义相对论还是圈量子引力 (LQG)，都没有否定空间作为一种实体存在的牛顿式直觉。在这些理论中，牛顿空间只是被更好地理解为——经典或量子 (分别对应上述两种理论)——动力学场的近似描述。相对于该场，位置可以通过关系方式定义，就像位置可以相对于任何其他事物通过关系方式定义一样。

In LQG, the spinnetwork states form a basis of states for the quantum gravitational field. (Later on we shall be more precise about the meaning of ”state” in this context.) A spinnetwork state $|\Gamma, j_\ell, v_n\rangle$ is determined by a labeled (abstract, not-embedded) graph Γ with links ℓ labeled by spins j_ℓ and nodes n labeled by $SU(2)$ intertwiners v_n . These are a basis of $SU(2)$ invariant tensors in $H_n = \otimes_{\ell \in n} V_{j_\ell}$, where the tensor product is over the links ℓ adjacent to the node n and V_j is the Hilbert space of the spin- j representation of $SU(2)$. The nodes of the graph describe elementary quantum excitations or ”elementary quanta” of the gravitational field. (The interpretation and the physical reasons for these discrete structures are discussed later on, in section ”Physical Discreteness.”)

在圈量子引力 (LQG) 中，自旋网络态构成了量子引力场的态基。(后续我们会明确“态”在该语境下的具体含义。) 一个自旋网络态 $|\Gamma, j_\ell, v_n\rangle$ 由一张带标记的 (抽象的、非嵌入的) 图 Γ 确定: 其中链 ℓ 由自旋 j_ℓ 标记，节点 n 由 $SU(2)$ 不变张量空间中的 $SU(2)$ intertwiners(缠绕元) v_n 标记。这些是 $H_n = \otimes_{\ell \in n} V_{j_\ell}$ 中 $SU(2)$ 不变张量的一组基，张量积覆盖与节点 n 相邻的所有链 ℓ ，且 V_j 是 $SU(2)$ 的自旋- j 表示的希尔伯特空间。图的节点描述引力场的基本量子激发，即“基本量子”。(这些离散结构的阐释和物理依据会在后续“物理离散性”一节讨论。)

The links of the graph define a notion of contiguity between two nodes linked by the link. The notion of contiguity between the elementary quanta determines a spatial structure, in the sense of relational space. Therefore, space, in the relational sense, is present at the foundations of the theory. In other words, the quanta represented by the nodes of the graph are spatially located with respect to one another. Notice that they are not located - in any sense - into an external container space. In a slogan, they are not in space; rather, they themselves, with their contiguity relations, make up a relational space.

图的连线定义了被连线连接的两个节点之间的邻接概念。基本量子之间的邻接概念以关系空间的意义确定了空间结构。因此，关系意义上的空间是该理论的基础。换句话说，由图中节点表示的量子彼此之间具有相对空间位置。请注意，它们 (无论从哪种意义上讲) 都不位于外部容器空间中。简言之，它们并不存在于空间之中；相反，它们本身连同彼此的邻接关系共同构成了关系空间。

When matter is present [1-3], its degrees of freedom are defined as labels on the same graphs Γ as the gravitational field states; therefore, the notion of contiguity is equally defined with respect to other quanta as well. That is, quanta of space and other elementary quantum field excitations are all located with respect to one another, defining a relational spatial structure.

当物质存在时 [1-3]，物质的自由度被定义为与引力场态处于同一图 Γ 上的标记；因此，邻接概念对其他量子而言也同样成立。也就是说，空间量子与其他基本量子场激发都彼此相对定位，共同定义了一个关系性空间结构。

Confusion between these two different meanings of ”space” is at the root of a common misunderstanding of non-perturbative quantum gravity. Some authors, for instance, suggest that physics is inconsistent unless it is formulated in terms of observables ”located in space” [8] taking for granted that a notion of space as a container is necessary for understanding science. This assumption has no basis.

对“空间”这两种不同含义的混淆，是人们普遍误解非微扰量子引力的根源。例如，部分学者认为，物理学除非以“定位于空间中”[8] 的可观测物来表述，否则就是不自洽的，他们理所当然地认为，作为容器的空间概念是理解科学的必要前提。这一假设毫无依据。

We locate our observations with respect to one another (both spatially and temporally), but the idea that there should be a container space within which observations are located is only a useful theoretical construct well utilized by Newton, not a requirement for intelligibility. For millennia, before Newton, humankind found the world perfectly conceivable in terms of relative localization and not in terms of localization ”in space.” In fact, even ”location” in everyday usage usually refers to adjacency to some material object, for instance a location on Earth, and not a location with respect to an abstract unobservable entity such as Newtonian space. To claim that science is unintelligible without a container space is to fail to understand the possibility of a conceptualization of the world different from the Newtonian one. (See also the discussion in [9].)

我们彼此参照 (在空间与时间上) 定位观测结果，但认为存在一个容纳所有观测位置的容器空间，不过是牛顿充分利用的有用理论构造，而非认知可理解性的必然要求。数千年来，牛顿之前的人类完全可以通过相对定位理解世界，而非依靠“在空间中”的定位。实际上，即便在日常用语中，“位置”通常也指相对于某个物质实体的邻接关系——比如地球上的某个位置，而非相对于牛顿空间这种抽象不可观测实体的位置。声称没有容器空间科学就无法理解，是没能意识到还存在不同于牛顿体系的世界观概念化可能。(另见文献 [9] 中的讨论。)

The Emergence of the Continuous Metric Space

连续度量空间的涌现

Let us consider the third of the above approximations in some detail. The way a continuous metric geometry emerges from the spinnetwork states is similar to the way a continuous electromagnetic field emerges from the photon states of quantum electrodynamics. A continuous (intrinsic) 3d Riemannian geometry g can be approximated arbitrarily well by a 3d Regge triangulation formed by flat tetrahedra τ_n connected by triangles t_ℓ . The (two skeletons of the) dual of this triangulation defines a graph Γ . Its geometry can be captured by the variables

让我们详细考察上述近似中的第三种。连续度量几何从自旋网络态中涌现的方式，类似于连续电磁场从量子电动力学的光子态中涌现的方式。连续 (内蕴) 三维黎曼几何 g 可以由由平坦四面体 τ_n 通过三角形连接构成的三维里奇三角剖分 t_ℓ 任意精确地逼近。该三角剖分的对偶 (的两个骨架) 定义了一个图 Γ ，它的几何可以由变量刻画

$$G_n^{\ell\ell'} = A_\ell A_{\ell'} \mathbf{n}_\ell \cdot \mathbf{n}_{\ell'} \tag{1}$$

where A_ℓ are the areas and $\mathbf{n}_\ell$ the unit normals to the face of the tetrahedron τ_n . The operators

其中 A_ℓ 是面积，$\mathbf{n}_\ell$ 是四面体 τ_n 面的单位法向量。算符

$$\hat{G}_n^{\ell\ell'} \left|\Gamma, j_\ell, v_n\right\rangle = (\hbar G)^2 \left|\Gamma, j_\ell, \mathbf{E}_n^\ell \cdot \mathbf{E}_n^{\ell'} v_n\right\rangle \tag{2}$$

where the vector operators $\mathbf{E}_n^\ell$ are $SU(2)$ generators on the V_{j_ℓ} tensor component of H_n , are defined on the Hilbert space $\mathcal{H}_\Gamma$ spanned by the spinnetwork states on Γ . It can be shown that there are ”intrinsic” coherent states $\left|\psi_g\right\rangle \in \mathcal{H}_\Gamma$ such that

其中矢量算符 $\mathbf{E}_n^\ell$ 是 H_n 的 V_{j_ℓ} 张量分量上的 $SU(2)$ 生成元，定义在由 Γ 上的自旋网络态张成的希尔伯特空间 $\mathcal{H}_\Gamma$ 上。可以证明，存在“内蕴”相干态 $\left|\psi_g\right\rangle \in \mathcal{H}_\Gamma$ 满足

$$\left\langle\psi_g\left|\hat{G}_n^{\ell\ell'}\right|\psi_g\right\rangle = G_n^{\ell\ell'} + O(\hbar) \tag{3}$$

and the variance of these operators goes to zero with $\hbar$ [10-12].

且这些算符的方差随 $\hbar$ 趋于零 [10-12]。

Similarly, the extrinsic geometry of a Riemannian 3d space embedded in a 4d spacetime can be approximated by the an extrinsic geometry k of a triangulated 3d space. This is captured by the $4d$ dihedral angles θ_ℓ between normals to the tetrahedra; $\mathcal{H}_\Gamma$ carries a corresponding operator $\hat{\theta}_\ell$ [13, 14] and it can be shown that there are (minimally spread) ”extrinsic” coherent states $\left|\psi_{g,k}\right\rangle \in \mathcal{H}_\Gamma$ that satisfy the last equation as well as

类似地，嵌入四维时空的三维黎曼空间的外蕴几何可以由三角剖分三维空间的外蕴几何 k 逼近。这由四面体法向量之间的 $4d$ 二面角 θ_ℓ 刻画；$\mathcal{H}_\Gamma$ 对应一个算符 $\hat{\theta}_\ell$ [13, 14] ，且可以证明存在 (最小弥散)“外蕴”相干态 $\left|\psi_{g,k}\right\rangle \in \mathcal{H}_\Gamma$ ，它满足上一方程，同时也满足

$$\left\langle\psi_{g,k}\left|\hat{\theta}_\ell\right|\psi_{g,k}\right\rangle = \theta_\ell + \mathcal{O}(\hbar). \tag{4}$$

In simpler words: there are quantum states of LQG that approximate Riemannian intrinsic and extrinsic geometries in the classical limit, in the usual sense in which states of any quantum theory approximate configurations of its classical limit.

简而言之: 圈量子引力中存在在经典极限下逼近黎曼内蕴与外蕴几何的量子态，这符合任意量子理论中态逼近经典极限构型的常规定义。

Furthermore, there is some evidence (see below) that the LQG transition amplitudes define a dynamics that approximates pseudo-Riemannian geometries that solve the Einstein equation, arbitrary well. In this sense, the pseudo-Riemannian spacetime of classical general relativity can ”emerge” from quantum gravity: it is the standard sense in which classical trajectories ”emerge” from a quantum theory.

此外，已有证据 (见下文) 表明，圈量子引力的跃迁振幅定义的动力学可以任意精确地逼近满足爱因斯坦方程的伪黎曼几何。在这个意义上，经典广义相对论的伪黎曼时空可以从量子引力中“涌现”：这完全符合经典轨迹从量子理论中“涌现”的常规含义。

Various aspects of the Newtonian notion of space are lost when moving to more general frameworks: the special relativistic framework loses the notion of a preferred space foliation which is present in Newtonian spacetime; the general relativistic framework loses the notion of a metric structure independent from the dynamical degrees of freedom; the full quantum gravitational framework lose the notion of continuous physical space. These notions are useful within approximations, but are not appropriate to describe nature in a full quantum gravitational regime. This is not a problem because they are not needed to have a coherent and intelligible conceptual picture of reality.

牛顿空间概念的诸多特征在推广到更通用框架时会消失: 狭义相对论框架抛弃了牛顿时空中原有的优先空间分层概念；广义相对论框架抛弃了独立于动力学自由度的度量结构概念；完整量子引力框架抛弃了连续物理空间的概念。这些概念在近似下仍然有效，但不适用于描述完整量子引力区域的自然。这并非问题，因为我们不需要它们就能得到一致、清晰的实在概念图景。

On the other hand, the relational notion of space defined by contiguity of dynamical entities, which is the familiar one we use when talking about space in our everyday life, remains well defined in quantum gravity. Hence, certain aspects of the intuitive notion of space (continuity, space as a container, Riemannian geometry, etc.) are emergent, others (relational space) are not. Those that emerge emerge from the dynamics of the spinnetworks (states of the quantum gravitational field), in a way which is to a large extent similar to the way a continuous electromagnetic field emerges from discrete photons (states of the quantum electromagnetic field).

另一方面，由动力学实体邻接关系定义的关系性空间概念——也就是我们日常生活中谈论空间时所使用的熟悉概念——在量子引力中仍然是良定义的。因此，直观空间概念的某些特征 (连续性、作为容器的空间、黎曼几何等) 是涌现的，另一些 (关系性空间) 则不是。那些涌现的特征从自旋网络 (量子引力场的态) 的动力学中涌现，其过程在很大程度上类似于连续电磁场从离散光子 (量子电磁场的态) 中涌现的方式。

In a full gravitational regime, the metric structure of spacetime displays the typical quantum features. The most prominent of these are:

在完整引力区域中，时空的度量结构展现出典型的量子特性，其中最突出的是:

(i) The granularity implied by the discrete spectrum of the $\hat{G}_n^{\ell\ell'}$ operators [15]. This is the most distinct feature and the key result of LQG [16]. (On a possible discreteness of time see [17] and on a possibility of actually measuring it see [18].)

(i) 由 $\hat{G}_n^{\ell\ell'}$ 算符的离散谱所隐含的粒度 [15]。这是 LQG 最显著的特征与核心结果 [16]。(关于时间可能存在的离散性见 [17]，关于实际测量该离散性的可能性见 [18]。)

(ii) The fact that the geometry can be in quantum superposition of states with definite geometrical properties, with the usual characteristic phenomena such as interference and entanglement. On the possibility of testing this phenomenon (already predicted by non-relativistic quantum gravity), see [19].

(ii) 几何可以处于具有确定几何性质的态的量子叠加中，存在干涉、纠缠这类常见特征现象。关于检验该现象 (已被非相对论量子引力预言) 的可能性，见 [19]。

(iii) The short scale fuzziness due to the fact that the various operators defining geometry do not commute. Even the $\hat{G}_n^{\ell\ell'}$ operators do not all commute with one another [20] and hence cannot be diagonalized together: this fact determines the quantum fuzziness of the (intrinsic) 3d geometry at short scale.

(iii) 由于定义几何的各类算符不对易产生的短尺度模糊性。即使是 $\hat{G}_n^{\ell\ell'}$ 算符，也并非全部彼此对易 [20]，因此无法同时对角化: 这一性质决定了短尺度下 (内蕴) 三维几何的量子模糊性。

We close this section addressing two issues raised in the philosophy literature [9]. The first regards the interpretation of the states that are superpositions of spinnetworks with different graphs. How is contiguity well defined if there is more than one graph, and two graphs define a different notion of contiguity? The answer to this question is in the overall structure of the LQG Hilbert space. A Hilbert space $\mathcal{H}_{\Gamma'}$ spanned by the spinnetworks with given abstract graph Γ' is a (proper) subspace of any Hilbert space $\mathcal{H}_\Gamma$ where Γ' is a subgraph of Γ . Any specification of a superposition of states with different graphs Γ' and Γ'' must be written as a state in a state space $\mathcal{H}_\Gamma$ where both Γ' and Γ'' are subgraphs of Γ . (See section ”Truncation, Finite Graphs, and Finite Spinfoams”). Unless otherwise specified, the quantum superposition between two states with different graphs must be interpreted as a state in the Hilbert space $\mathcal{H}_\Gamma$ where Γ is formed by the two disconnected components Γ' and Γ'' . This resolves any ambiguity.

我们在本节最后讨论哲学文献中提出的两个问题 [9]。第一个问题关乎对不同图自旋网络叠加态的解释: 如果存在不止一个图，且两个图对邻接性的定义不同，那邻接性如何得到良好定义？这个问题的答案藏在 LQG 希尔伯特空间的整体结构中。由给定抽象图 Γ' 张成的希尔伯特空间 $\mathcal{H}_{\Gamma'}$，是任意希尔伯特空间 $\mathcal{H}_\Gamma$ 的真子空间，其中 Γ' 是 Γ 的子图。任何对不同图 Γ' 与 Γ'' 态叠加的描述，都必须写成态空间 $\mathcal{H}_\Gamma$ 中的一个态，且 Γ' 和 Γ'' 都是 Γ 的子图。(参见“截断、有限图与有限自旋泡沫”一节。) 除非另有说明，两个不同图的态之间的量子叠加，必须解释为希尔伯特空间 $\mathcal{H}_\Gamma$ 中的态，其中 Γ 由两个不连通分支 Γ' 和 Γ'' 构成。这就解决了所有歧义。

The second issue raised in [9] is the observation that the notion of adjacency defined by the graph may not match the one implicitly defined by an averaged large geometry. Hence, there may be two distinct notions of contiguity in the theory: the one defined by the graph (that underpins the dynamics of the spinnetworks) and the one defined by the emergent smooth geometry. This is correct. The same happens in classical general relativity. A wormhole smaller than the scale of observation can connect macroscopically distant regions of spacetime. A microscopic notion of contiguity does not need to match the macroscopic one. There can be ”wild” geometries in classical general relativity, where a similar mismatch happens, and this does not jeopardize intelligibility (In addition, regular geometries can be discretized in terms of ”wild” triangulations. See later.). Similarly, there are ”wild” states in LQG. We do not know the physical relevance (if any) of either.

文献 [9] 提出的第二个问题指出，图定义的邻接概念可能与平均大尺度几何隐式定义的邻接概念不匹配。因此该理论中可能存在两种不同的邻接概念: 图定义的邻接 (是自旋网络动力学的基础) 和涌现光滑几何定义的邻接。这个观察是正确的。同样的情况也存在于经典广义相对论中: 一个小于观测尺度的虫洞可以连接宏观上相距遥远的时空区域。微观层面的邻接概念不需要和宏观层面的匹配。经典广义相对论中也可以存在“非常规”几何，出现类似的不匹配，但这并没有损害理论的可理解性 (此外，规则几何也可以用“非常规”三角剖分离散化，见后)。类似地，LQG 中也存在“非常规”态。我们目前不知道二者 (如果存在的话) 的物理相关性。

To clarify why two notions of adjacency do not represent a problem, it is useful to ask what the physical meaning of adjacency is. The answer should not be searched in a Kantian a priori condition for intelligibility, but in what we have learned from experience about the world around us. The adjacency relation that we experience is rooted in the fact that physical interactions are local. Because of this, we only directly affect - and we only directly receive information from - adjacent entities. In other words, the basic spatial structure of the world is determined by what directly affects what. The dynamics of loop quantum gravity is local on the graph (both in the Hamiltonian and in the covariant formulations of LQG). This is why the locality relation that we experience derives from the locality defined by the graph structure of the states.

为了说明为什么两种邻接概念不成问题，我们不妨追问邻接的物理意义是什么。我们不应当从康德式的可理解性先天条件中寻找答案，而应当从我们对周遭世界的经验所得中寻找。我们体验到的邻接关系，根植于物理相互作用是局域的这一事实。正因如此，我们只能直接影响相邻实体，也只能直接从相邻实体获取信息。换句话说，世界的基本空间结构由“什么直接影响什么”决定。圈量子引力的动力学在图上是局域的 (无论是在 LQG 的哈密顿表述还是协变表述中)。这就是我们体验到的局域性关系来源于态的图结构所定义的局域性的原因。

Observability in Gravitational Physics

引力物理学中的可观测性

The conceptual structure of general relativity (GR) is subtle and has confused all relativists (including Einstein) for a long time. For this reason, decades have been necessary before getting clarity on question like the nature of the Schwarzschild singularity or whether gravitational waves are physical or gauge artifacts.

广义相对论 (GR) 的概念结构十分微妙，长期以来困惑了所有相对论研究者 (包括爱因斯坦本人)。正因如此，人们花费了数十年时间才弄清楚诸如史瓦西奇点的本质、引力波是物理实体还是规范伪影这类问题。

Much of the confusion stems from the fact that the theory is written in terms of spacetime coordinates x and t , but the physical meaning of these is totally different from the physical meaning of the spacetime coordinates with the same name used in special relativity and in non-relativistic physics. The spacetime coordinates X and T in non-relativistic and special relativistic physics have metric meaning: the spacetime coordinates x and t in general relativistic physics do not have metric meaning. That is, in special relativity for a particle to have position X means to be at a physical distance X from the axis of some established physical reference frame. This distance can be measured with a rod, a laser, or anything else. For an event to have coordinate T means to happen when a clock has measured a time lapse T . Not so in GR: in GR, if a particle has position x , this does not mean that the particle is at a physical distance x from something. For an event to have coordinate t does not mean that a clock has measured a time lapse t from some initial time. Distance (measured in any of the above manners) and clock readings are rather given by integrals involving the gravitational field, such as

> 大部分困惑都源于一个事实: 该理论以时空坐标 x 和 t 表述，但这些坐标的物理意义，与狭义相对论和非相对论物理学中同名时空坐标的物理意义完全不同。非相对论物理和狭义相对论中的时空坐标 X 和 T 具有度规意义: 广义相对论中的时空坐标 x 和 t 不具有度规意义。也就是说，在狭义相对论中，一个粒子的位置为 X ，意味着它距离某个已建立的物理参考系的轴的物理距离为 X ，这个距离可以用尺、激光或其他工具测量。一个事件的坐标为 T ，意味着它发生时，时钟已经测量到经过了时间间隔 T 。广义相对论并非如此: 在广义相对论中，若一个粒子的位置为 x ，这不代表它距离某个物体的物理距离为 x ；一个事件的坐标为 t ，也不代表时钟测量到从某个初始时刻起经过了时间间隔 t 。距离 (以上述任意方式测量) 和时钟读数，实际上由包含引力场的积分给出，例如

$$T = \int_\gamma \sqrt{g_{ab} dx^a dx^b}, \tag{5}$$

where $g_{ab}(x)$ is the gravitational field. The value of these integrals does not change if the coordinates are changed to new coordinates. The fact that the coordinates have such a dramatically different meaning in the two contexts raises continuous confusion.

> 其中 $g_{ab}(x)$ 是引力场。若将坐标更换为新坐标，这些积分的值不会改变。坐标在两种语境下的含义差异如此之大，持续引发着困惑。

Related to this is a persistent confusion about the connection between the theory and the physical measurable quantities, usually called ”observables.” The reason of the confusion is the following. The Einstein equations are invariant under arbitrary changes of the coordinates x and t . It follows that in general a quantity that depends on the coordinates x and t cannot be predicted by the theory.

> 与此相关的是，人们一直对理论与通常称为“可观测量”的物理可测量量之间的关联存在困惑。困惑的原因如下: 爱因斯坦方程在坐标 x 和 t 的任意变换下保持不变，由此可得，依赖于坐标 x 和 t 的量一般无法被理论预测。

There are three alternative ways of interpreting this fact and using the theory, all three are equally valid (see [21]).

> 对这一事实和该理论的使用有三种不同的解读方式，三种都同等有效 (参见文献 [21])。

1. The first is to only consider observables that are invariant under coordinate transformations. These are predicted by the Einstein equations, once a solution is specified. For instance, the minimal distance between the Earth and the Moon during the current month, as measured by the return (proper-)time on Earth of a laser signal bounced off the moon, is a quantity that does not depend on the coordinates chosen. All quantities measured in relativistic observational gravity can be interpreted in this manner.

1. 第一种是仅考虑在坐标变换下不变的可观测量。一旦给定解，这些可观测量就可以被爱因斯坦方程预测。例如，本月地月之间的最小距离——通过测量月球反射激光信号返回地球的 (固有时) 时间得到——就是一个不依赖所选坐标的量。相对论引力观测中测量的所有量都可以用这种方式解读。

2. The second option is to interpret the coordinates as labels of concrete reference objects whose dynamics is determined by the theory. This is a gauge fixing of the coordinate choice, and as such it promotes the coordinates to quantities that can be actually determined physically. This procedure is commonly followed for instance in cosmology (in the homogeneous approximation), where coordinates are attached to galaxies and the proper time on these. In this language, the Einstein equations are gauge fixed on particular coordinate choices.

2. 第二种方式是将坐标解读为具体参考对象的标记，这些参考对象的动力学由理论决定。这是对坐标选择的规范固定，借此将坐标提升为可实际通过物理测量确定的量。该方法被普遍应用于宇宙学研究中 (均匀近似下)，在那里坐标被绑定到星系以及这些星系上的固有时。按照这种表述，爱因斯坦方程在特定坐标选择下完成规范固定。

3. The third option is again to interpret the coordinates as labels of concrete reference objects but to disregard the dynamical laws governing these reference objects. The under-determination in the evolution equations can then be interpreted as the result of disregarding these dynamical laws, namely, choosing physical reference systems that move arbitrarily in spacetime.

3. 第三种方式同样将坐标解读为具体参考对象的标记，但忽略支配这些参考对象的动力学定律。演化方程中的不确定性，可以被解读为忽略这些动力学定律的结果，也就是选择了在时空中任意运动的物理参考系。

The three options are all viable and ultimately equivalent. They refer to different sets of variables. While the first and the second refer to the physics of the dynamical degrees of freedom included in the theory, with nothing else interacting, the third refers to the physics of the dynamical degrees of freedom of the theory interacting with other degrees of freedom.

这三种方式都可行，且最终等价。它们对应不同的变量集合: 第一种和第二种对应理论本身包含的动力学自由度的物理，不存在其他外界相互作用；第三种则对应理论的动力学自由度与其他自由度相互作用的物理。

In other words, the gauge degrees of freedom of general relativity can alternatively be:

换句话说，广义相对论的规范自由度可以被归类为:

1. Interpreted as unphysical, namely, just as a redundancy of the mathematics

1. 被解读为非物理的，即仅仅是数学表述的冗余

2. Gauge fixed

2. 进行规范固定

3. Interpreted as relational degrees of freedom describing the coupling with an external (arbitrarily moving) physical system, used as physical reference system [22]

> 3. 被解读为关联自由度，描述与被用作物理参考系的外部 (任意运动的) 物理系统的耦合 [22]

In all interpretations, spacetime localization is only relative. In the first case, objects and events in the theory are localized with respect to one another (the Earth and the laser pulse). In the second, they are localized with respect to the chosen reference system (for instance, the galaxies and their clocks). In the third, they are localized with respect to the external arbitrarily moving reference system. Much of the conceptual confusion about the observables of general relativity and about the interpretation of ”spacetime points” comes from mixing up these three cases.

> 在所有诠释中，时空局域化都只是相对的。第一种情况里，理论中的物体与事件是相对彼此定位的 (地球与激光脉冲)。第二种情况里，它们是相对所选参考系定位的 (例如星系与其时钟)。第三种情况里，它们是相对外部任意运动的参考系定位的。广义相对论可观测量与“时空点”诠释的大量概念困惑，都源于混淆了这三种情况。

A consideration is important for what follows. Are observables in general relativity local in some sense? Let us consider two examples taken from actual applications of the theory, where physical quantities are concretely measured by experimentalists and astronomers, and compared to the theory. As a first example, consider a detection of a gravitational wave pulse by a gravitational interferometer. This can be thought as a curvature measurement in a location defined by components of the detector. It is local in the sense that it only involves what happens in the location of the detector. A second example is a typical measurements in the analysis of the general relativistic dynamics of the solar system. Astronomers measure the physical distance between a given point on Earth and a given point on Venus, at a specific time determined by some event of Earth, and defined as half the forward-backward travel time of a laser pulse that bounces off Venus, where the travel time is in terms of the Earth proper time. The measured quantity is local in the sense that it only involves what happens in the solar system. Now, say we interpret the theory according to the first option above. Are the actual measured diffeomorphism invariant variables local, in the sense that they can be expressed as local functions of the coordinates in the dynamical system formed by all the entities involved? Of course they are not, because no diffeomorphism invariant quantity is local in this sense. These examples should lead us to caution: one often reads that the absence of ”local” observables represents a major obstacle in interpreting general relativity and quantum gravity. This is certainly not the case in the classical theory, as shown by these examples. We shall come back on this in the quantum context.

接下来的讨论有一个要点需要说明。广义相对论中的可观测量在某种意义上是局域的吗？我们来看两个取自该理论实际应用的例子，其中物理量由实验学家和天文学家具体测量，并与理论比对。第一个例子是引力干涉仪探测引力波脉冲，这可以理解为在探测器组件定义的位置进行曲率测量。它是局域的，因为它只涉及探测器位置发生的过程。第二个例子是太阳系广义相对论动力学分析中的典型测量: 天文学家测量地球某一点与金星某一点在地球某一事件确定的特定时刻的物理距离，这个时刻定义为激光脉冲弹回金星往返行程时间的一半，行程时间以地球固有时计算。该测量量是局域的，因为它只涉及太阳系内发生的过程。现在，假设我们按照上述第一种选项诠释该理论: 实际测量的微分同胚不变量是局域的吗？也就是说，它们能否表示为所有相关实体构成的动力学系统中坐标的局域函数？答案当然是否定的，因为没有任何微分同胚不变量在这个意义上是局域的。这些例子提醒我们要注意: 人们常说"局域"可观测量的缺失是广义相对论和量子引力诠释的主要障碍。但正如这些例子所示，在经典理论中情况显然并非如此，我们会在量子背景下再回到这个问题。

Notice that if we adopt the third reading of general covariance, the interpretation of what is measured in the two examples above simplifies dramatically. Take the case of the detection of the gravitational wave pulse. In the first interpretation, we consider the coupled dynamical system formed by the gravitational field and the interferometer, and the measured quantities are a highly nonlocal function of the basic variables. In the third interpretation, instead, we can think that the system under consideration is just the gravitational field and view the laboratory containing the detector as an external (”reference”) system with which the gravitational field is interacting. The quantity measured by the detector is a local function of the metric, in the location determined by the detector. The full diffeomorphism invariance of the pure gravity dynamics, in other words, is physically broken by the detector itself being located somewhere.

注意，如果我们采用广义协变性的第三种解读，上述两个例子中测量对象的诠释会大幅简化。以引力波脉冲探测为例: 在第一种诠释中，我们考虑的是引力场与干涉仪构成的耦合动力学系统，测量量是基本变量的高度非局域函数。而在第三种诠释中，我们可以认为研究系统只是引力场，容纳探测器的实验室是与引力场相互作用的外部"参考"系统。探测器测量的量是探测器所在位置度规的局域函数。换句话说，纯引力动力学的完整微分同胚不变性，被位于某处的探测器本身从物理上破缺了。

The two interpretations are equally correct, and both have advantages. As we shall see, the second opens up an interesting window of opportunity in the quantum context, in relation to the necessity of considering a Heisenberg cut in quantum measurements.

这两种诠释同样正确，且各有优势。正如我们将会看到的，第二种诠释在量子背景下打开了一个有趣的机会窗口，这和量子测量中必须考虑海森堡切分有关。

Mixing up three interpretations above is also the source of the confusion in the discussion about the meaning of spacetime points in general relativity (and the ”hole argument” [23]) which has been going on in philosophy of science [24]. The discussion is confused by the fact that in the first interpretation there is no physical definition of points independently from the degrees of freedom of the theory, but in the third there is such definition (because the points, individuated by the coordinates, are defined relationally with respect to the external arbitrarily moving reference system).

混淆上述三种诠释，也是广义相对论时空点意义 (以及“孔洞论证”[23]) 相关讨论产生困惑的根源，相关讨论已经在科学哲学领域持续多年 [24]。讨论之所以混乱，是因为在第一种诠释中，不存在独立于理论自由度的点的物理定义，但在第三种诠释中存在这样的定义 (因为由坐标标识的点是相对外部任意运动参考系关系性定义的)。

The third of the above interpretations is the reason for the strong (irresistible, for some) intuitive appeal of the reality of a manifold independent from the value of the gravitational field defined over it. The points of the manifolds are possibilities for coupling other degrees of freedom. Once we include all degrees of freedom, the manifold is dispensable (as many relativists like to repeat [25]). The same is true for the graph of a spinnetwork and the two complexes of a spinfoam, if these are considered independently from their labeling.

上述第三种诠释，就是独立于其上定义的引力场的流形实在拥有强烈 (对某些人来说无法抗拒) 直觉吸引力的原因。流形上的点是耦合其他自由度的可能性。一旦我们包含所有自由度，流形就是多余的 (正如许多相对论学家反复强调的 [25])。对于自旋网络图和自旋泡沫二复形而言，如果脱离它们的标记单独考虑，情况也是如此。

This discussion, more broadly, also sheds light on the general interpretation of gauge invariance: gauge is more than mathematical redundancy, because the gauge degrees of freedom capture ways a physical system can couple with other physical systems. This is because (gauge-invariant) couplings can couple to gauge variant variables of a component system. A discussion on this fact is in [22].

更广泛来说，这一讨论也阐明了规范不变性的一般诠释: 规范不只是数学冗余，因为规范自由度描述了物理系统与其他物理系统耦合的方式。这是因为 (规范不变的) 耦合可以与分量系统的规范可变量耦合。相关讨论可见 [22]。

General Relativistic Evolution

广义相对论演化

Physics describes processes, namely, how things happen or how they ”change.” To do so, general relativistic physics employs a more subtle notion of evolution than Newtonian physics.

物理学研究过程，即事物如何发生、如何“变化”。为此，广义相对论物理学采用了比牛顿物理学更精妙的演化概念。

In Newtonian physics, evolution is described by writing equations that govern how physical variables change in time. In general relativistic physics, dynamical processes are described by writing equations that govern how physical variables (including those characterizing clocks) change with respect to one another [26-29]. A characteristic example of utilization of this relative notion of evolution is loop quantum cosmology [30] where the dynamics of the universe is often coded in the relative evolution between the cosmological scale factor and the value of a homogeneous scalar field.

在牛顿物理学中，演化通过描述物理变量如何随时间变化的方程来刻画。在广义相对论物理学中，动力学过程通过描述物理变量 (包括表征时钟的变量) 如何相互关联变化的方程来刻画 [26-29]。圈量子宇宙学 [30] 就是采用这种相对演化概念的典型例子，在该理论中，宇宙的动力学通常编码在宇宙标度因子与均匀标量场取值之间的相对演化中。

More precisely, in Newtonian physics we use dynamical variables $A, B, \ldots$ plus a "special" (preferred, canonical, etc.) time variable T . We call "clocks" the measuring devices that best track this variable. The time variable T is used as the independent variable of the evolution, and we write equations of motion for the functions $A(T), B(T), \ldots$. A motion can equally be represented as a line in the space of the variables $(T, A, B, \ldots)$, defined implicitly ("covariantly") by functions of the form $f(T, A, B, \ldots) = 0$.

更准确地说，在牛顿物理学中，我们使用动力学变量 $A, B, \ldots$ 加上一个“特殊”(优先、正则等) 时间变量 T 。我们将最能追踪该变量的测量装置称为“时钟”。时间变量 T 被用作演化的自变量，我们为函数 $A(T), B(T), \ldots$ 写下运动方程。运动同样可以表示为变量 $(T, A, B, \ldots)$ 空间中的一条线，由形式为 $f(T, A, B, \ldots) = 0$ 的函数隐式 (“协变地”) 定义。

In a general relativistic physics, evolution is described by writing equations that govern how physical variables (including those measured by clocks) change with respect to one another. This is because there is no single canonical time variable. Different "clocks" determine distinct measurable variables $T_n, n = 1, 2, \ldots$ Accordingly, we define evolution by giving relations between all variables including the clocks. A motion is therefore described by a line in the space spanned by all variables $(T_n, A, B, \ldots)$., defined covariantly by functions of the form $f(T_n, A, B, \ldots) = 0$. This line can be parametrized by an arbitrary label t : this is the relativistic time coordinate, which should not be confused with the readings T_n of clocks.

在广义相对论物理学中，演化通过描述物理变量 (包括时钟测量的变量) 如何相互关联变化的方程来刻画。这是因为广义相对论中不存在单一的正则时间变量。不同的“时钟”对应不同的可测量变量 $T_n, n = 1, 2, \ldots$ 。相应地，我们通过给出包括时钟在内所有变量之间的关系来定义演化。因此，运动可以用所有变量 $(T_n, A, B, \ldots)$ 张成空间中的一条线来描述，该线由形式为 $f(T_n, A, B, \ldots) = 0$ 的函数协变定义。这条线可以用任意参数 t 参数化: 这个参数就是相对论时间坐标，注意不要将它与时钟的读数 T_n 混淆。

The above is easily generalized to field theory. In a 4d (non-general-relativistic) field theory, evolution is described by equations for fields $A(X, T), B(X, T), \ldots$ that depend on spacetime coordinates (X, T) . These coordinates represent distances measured by rods and time intervals measured by clocks. A motion can equally be represented as a $4d$ surface in the space spanned by the variables $(X, T, A, B, \ldots)$. In a general relativistic field theory like general relativity, on the other hand, evolution is described by writing equations that govern how physical variables (including distances measured by rods and time intervals measured by clocks) change with respect to one another. Evolution is given by relations between all variables including clock variables T_n and distance variable X_n . A motion is a 4d surface in the space of all variables $(X_n, T_n, A, B, \ldots)$. This surface can be parametrized by arbitrary 4 d labels (x, t) : these are the relativistic spacetime coordinates.

上述内容可以很容易推广到场论。在四维 (非广义相对论) 场论中，演化由依赖于时空坐标 (X,T) 的场 $A(X,T)$, $B(X,T)$, ... 的方程描述。这些坐标代表用尺测量的距离和用时钟测量的时间间隔。运动同样可以表示为变量 $(X,T,A,B,...)$ 张成空间中的一张 $4d$ 超曲面。另一方面，在类似广义相对论这样的广义相对论场论中，演化通过描述物理变量 (包括用尺测量的距离和用时钟测量的时间间隔) 如何相互关联变化的方程来刻画。演化由所有变量之间的关系给出，包括时钟变量 T_n 和距离变量 X_n 。运动是所有变量 $(X_n,T_n,A,B,...)$ 空间中的一张四维曲面。这张曲面可以用任意 4 d 坐标 (x,t) 参数化: 这些坐标就是相对论时空坐标。

The fact that there is no preferred time variable in relativistic gravitational physics can be seen for instance by noticing that in general two clocks measure different (proper) times between the same couple of events, depending on their location, speed, etcetera. Consider the following example: launch a clock C_1 upward at some moment and catch it back when it falls at a second moment. In the meanwhile, hold a second clock in your hands. The two clocks will measure two different times T_1 and $T_2 < T_1$ between the launch and the catch. Which one is the real time variable? The answer is that there is no "real" time : both times are physical times and can be taken as independent variables.

相对论引力物理中不存在优先时间变量这一点，可以从以下事实看出: 一般来说，两个时钟在同一对事件之间测量的 (固有时) 不同，测量结果取决于它们的位置、速度等。举个例子: 在某个时刻将一个时钟 C_1 向上抛出，在它下落回时再次接住它；在此过程中，你手持另一个时钟保持静止。两个时钟在抛出和接住之间会测量出不同的时间 T_1 和 $T_2 < T_1$ 。哪一个才是真实的时间变量？答案是不存在“真实”时间: 两个时间都是物理时间，都可以作为自变量。

The way evolution is treated in general relativistic physics is reflected in the Hamiltonian structure of the dynamical theory. The general structure of Newtonian physics is given by a Hamiltonian H on a phase space Γ . The phase space is a symplectic manifold. In a symplectic manifold, a function H generates a flow: the physical motions are the orbits generated by the flow of H in Γ .

广义相对论物理学中对演化的处理方式，反映在动力学理论的哈密顿结构中。牛顿物理学的一般结构由相空间 Γ 上的哈密顿量 H 给出。相空间是一个辛流形。在辛流形中，函数 H 会生成一个流: 物理运动就是 Γ 中由 H 的流生成的轨道。

General relativistic physics requires a generalization of this structure, which we sketch here (for more details, see [1]). The generalization is given by a constraint C on an extended (symplectic) phase space Γ_{ex} . The symplectic form on Γ_{ex} induces a pre-symplectic form on the constraints surface $C = 0$. The motions are the lines (surfaces in field theory) on the constraint surfaces whose tangents are null directions of the pre-symplectic form. The general (finite dimensional) case reduces to the Newtonian case when Γ_{ex} is the Cartesian product of Γ and a space with canonically conjugate coordinates (T, p_T) , and $C = H + p_T$, as can be easily verified.

广义相对论物理学要求对这一结构进行推广，我们在此概述其概要 (更多细节参见文献 [1])。该推广由扩广 (辛) 相空间 Γ_{ex} 上的一个约束 C 给出。Γ_{ex} 上的辛形式在约束面 $C = 0$ 上诱导出一个预辛形式。运动就是约束面上切线为预辛形式零方向的线 (场论中为曲面)。不难验证，当 Γ_{ex} 为 Γ 与带正则共轭坐标 (T, p_T) 和 $C = H + p_T$ 的空间的笛卡尔积时，一般有限维情况可约化为牛顿力学情况。

The quantities like $T, A, B, \ldots$, that include dependent as well as independent dynamical variables are called partial observables [31]. These are quantities that can be measured but cannot be individually predicted even with full knowledge of the motion (as they include the independent variables of the evolution). What the theory predicts is not the value of individual partial observables, but, rather, relations among them. For instance, for a harmonic oscillator with a single degree of freedom, its position X and the time T are both partial observables, and once the motion is known (amplitude and phase are known), the theory predicts the value of X for any given T , or the possible values of T for any given X .

像 $T, A, B, \ldots$ 这类同时包含依赖和独立动力学变量的量称为部分可观测量 [31]。这些量是可测量的，但即使完全掌握运动信息也无法单独预测其取值 (因为它们包含演化的独立变量)。理论预测的不是单个部分可观测量的值，而是它们之间的关系。例如，对于单自由度简谐振子，其位置 X 和时间 T 都是部分可观测量；当运动已知 (振幅和相位确定) 后，理论可以预测任意给定 T 对应的 X 值，或是任意给定 X 对应的可能 T 值。

The physical phase space is the space of the motions. It is again a symplectic space and in the case of a Newtonian theory is isomorphic to the familiar space of the initial data, but not canonically so, because isomorphism is determined by a value of T . Each point of the physical phase space determines a relation between partial observables.

物理相空间是运动的空间。它同样是一个辛空间，在牛顿理论中同构于我们熟悉的初值空间，但不是正则同构，因为同构由 T 的取值决定。物理相空间的每个点都确定了部分可观测量之间的一个关系。

The motions can be parametrized. That is, the surfaces defined by the functions $f(A_n)$, where A_n coordinatize the space of the partial observables of the theory, can be written as functions $A_n(x)$ of arbitrary coordinates x . The spacetime coordinates used in general relativity are these parameters.

运动可以参数化。也就是说，由函数 $f(A_n)$ 定义的曲面 (其中 A_n 为该理论部分可观测量空间的坐标) 可以写成任意坐标 x 的函数 $A_n(x)$ 。广义相对论中使用的时空坐标就是这类参数。

In quantum theory, the partial observables are represented by self-adjoint operators on an extended Hilbert space $\mathcal{H}_{\text{ex}}$ and the dynamics is given by a constraint operator (or set of operators) C defined on $\mathcal{H}_{\text{ex}}$. The transition amplitudes that define the quantum dynamics amplitudes are given by

在量子理论中，部分可观测量由扩广希尔伯特空间 $\mathcal{H}_{\text{ex}}$ 上的自伴算符表示，动力学由定义在 $\mathcal{H}_{\text{ex}}$ 上的约束算符 (或算符集) C 给出。定义量子动力学振幅的跃迁振幅由下式给出

$$W(a, b) = \langle a |P| b\rangle \tag{6}$$

where $|a\rangle$ and $|b\rangle$ are eigenstates of (a complete set of commuting) observables defined on $\mathcal{H}_{\text{ex}}.P$ is the projector of the kernel of C if this is a proper subspace of $\mathcal{H}_{\text{ex}}$. If zero is in the continuum spectrum, P can equally be defined using distributional techniques; see for instance [1, 32, 33] .

其中 $|a\rangle$ and $|b\rangle$ 是定义在 $\mathcal{H}_{\text{ex}}.P$ 上的 (对易可观测量完全集) 的本征态，若零空间是 $\mathcal{H}_{\text{ex}}$ 的真子空间，则 C 是其零空间的投影算子。若零位于连续谱中，P 同样可以利用分布技巧定义；参见例如 [1, 32, 33] 。

The transition amplitudes (6) define the quantum dynamics of a general covariant quantum field theory.

跃迁振幅 (6) 定义了广义协变量子场论的量子动力学。

To compute probabilities from the amplitudes we must remember that H_{ex} describe partial observables, which include the independent variables. We can therefore only assign probabilities to some components (say, a_1) of $a = (a_1, a_0)$ at a given value of others (a_0) . That is, probabilities are well defined when

要从振幅计算概率，我们必须记住 H_{ex} 描述的是包含独立变量的部分可观测量。因此我们只能对其他分量取给定值 (a_0) 时，$a = (a_1, a_0)$ 的某些分量 (例如 a_1) 赋予概率。也就是说，仅当满足下述条件时概率才有良好定义:

$$\sum_{a_1} |W(a_1, a_0, b)|^2 = 1. \tag{7}$$

If the set of variables a_o include a variable t such that the dynamics is symmetric under a translation in t , then the Hilbert space carries a unitary representation of the group $\mathbb{R}$. If there is no variable with this property, then there is no unitarity in this sense in the theory, but this does not mean that probabilities are ill defined. For an enlightening simple example of a well-defined quantum system without unitarity in this sense, see [34]. For a discussion on the definition of probability in the general case, see [35].

如果变量集 a_o 包含某个变量 t ，使得动力学在 t 的平移变换下具有对称性，那么希尔伯特空间承载了群 $\mathbb{R}$ 的一个幺正表示。如果不存在具备该性质的变量，那么理论在这个意义上就不存在幺正性，但这不意味着概率是没有良定义的。关于满足该定义、不具有这种意义上幺正性的量子系统的一个启发性简单例子，参见文献 [34]。关于一般情况下概率定义的讨论，参见文献 [35]。

Observability in Quantum Physics

量子物理学中的可观测性

Classical physics assumes that all properties of a system are always sharply defined. Not so quantum physics. Properties are given by eigenvalues of observables and quantum mechanics only assigns properties to a system in the context of an interaction with another system. The boundary between the two is called the Heisenberg cut [36]. For instance, in the Copenhagen interpretation properties are actualized in measurements; in the relational interpretation, they are always relative to a second system; in the many-world interpretations, they depend on the branching and are related to Everett's relative states determined by a split due to a Heisenberg cut (The terminology ”Heisenberg cut” is characteristic of the Copenhagen interpretation, but we use it more generally to denote the separation between systems which is needed in order to have actual values of variable also in interpretations such as many-worlds and relational.).

经典物理学假设一个系统的所有性质始终是清晰定义的。量子物理学并非如此。在量子力学中，性质由可观测对象的本征值给出，且只有在系统与另一系统发生相互作用时，才会为系统赋予性质。两个系统之间的分界被称为海森堡切割 [36]。例如，在哥本哈根诠释中，性质在测量中被实现；在关系性诠释中，性质始终是相对于第二个系统而言的；在多世界诠释中，性质依赖于分支，并且与由海森堡切割引发分裂所确定的埃弗雷特相对态相关 (“海森堡切割”这一术语是哥本哈根诠释的典型用语，但我们在此对它做更广义的使用，用来表示分隔系统的界限——即便在多世界、关系性这类诠释中，要让变量拥有实际值也需要这一分界)。

For simplicity, we use the language of the Copenhagen interpretation, but what we say can be easily translated in the language of different interpretations. In the Copenhagen interpretation we mentally distinguish the system from the context, separated by the Heisenberg cut and treat the context classically. The observables of the system take a value at a measurement, which is an interaction between the system and the context. The theory predicts the probability of one or the other value to be actualized in this interaction (called measurement), given that other values of observables were actualized in a previous interaction (called preparation). The cut can be moved outward without changing the predictions. Denoting a the set of the observables' values actualized in the preparation and b the set of the observables' values actualized in the measurement, the conditional probabilities predicted by the theory are given by

为简化表述，我们使用哥本哈根诠释的语言，但我们的论述可以很容易转换为不同诠释的语言。在哥本哈根诠释中，我们在概念上将系统与由海森堡切割分隔开的环境区分开，并将环境按经典方式处理。系统的可观测对象会在测量 (即系统与环境的相互作用) 中取得一个确定值。给定前一次相互作用 (称为制备) 中已经实现的可观测对象值，理论会预测本次相互作用 (称为测量) 中实现某个具体值的概率。海森堡切割可以向外移动，且不会改变预测结果。记 a 为制备过程中实现的可观测对象值的集合，b 为测量过程中实现的可观测对象值的集合，理论预测的条件概率由下式给出

$$P(b \mid a) = |W(b, a)|^2 \tag{8}$$

where the transition amplitude is given by

其中跃迁振幅由下式给出

$$W(b; a) = \langle b \mid a \rangle; \tag{9}$$

here $|a\rangle$ and $|b\rangle$ are the relevant normalized eigenstates of the operators corresponding to the relevant observables. (In the relational interpretation the cut defines relational observables; in the many-world interpretation, the cut separates two subsystems that define the branching, within which variables have determined values.)

此处 $|a\rangle$ and $|b\rangle$ 是对应相关可观测对象的算符的相关归一化本征态。(在关系性诠释中，切割定义了关系可观测对象；在多世界诠释中，切割分隔了两个子系统，二者共同定义分支，分支内部的变量拥有确定值。)

If a time $(t' - t)$ different from zero lapses between the two measurements and the Hamiltonian is H, the transition probabilities are

如果两次测量之间间隔了一个不为零的时间 $(t' - t)$，且哈密顿量为 H，则跃迁概率为

$$W(b, t'; a, t) = \left\langle b \left| e^{-\frac{i}{\hbar}(t'-t)H} \right| a \right\rangle; \quad (10)$$

In the general relativistic case, the time variable is included among the partial observables, and we write the amplitude above as

在广义相对论的情况下，时间变量被包含在部分可观测对象中，我们将上述振幅写为

$$W(b, t'; a, t) = \langle b, t' |P| a, t \rangle. \quad (11)$$

Additionally, we can consider a "boundary" Hilbert space $H_b = H_{in} \otimes H_{out}$, namely, the tensor product of the in and out state spaces, and express the dynamics as a single (possibly generalized) bra on this boundary state space:

此外，我们可以考虑一个“边界”希尔伯特空间 $H_b = H_{in} \otimes H_{out}$，即入射态空间和出射态空间的张量积，并将动力学表示为这个边界态空间上的单个 (可能是广义的) 左矢:

$$W(b, t'; a, t) = \langle W \mid b, t'; a, t \rangle. \quad (12)$$

For details on the formalism, see [1].

有关该形式体系的更多细节，参见文献 [1]。

Observability in Quantum Gravity

量子引力中的可观测性

To understand observability in quantum gravity we have to combine our understanding of observability in general relativity with our understanding of observability in quantum theory. From the second, we learn that what the theory can predict is the probability of one or the other property of the gravitational field to be actualized in interactions, across a Heisenberg cut, with a context that can be treated as classical. The natural and simple key for this to work is to identify the Heisenberg cut with the boundary of a four-dimensional spacetime region $\mathcal{R}$ [37]. It is not difficult to see that any realistic observation in relativistic gravitation can be expressed in this form.

要理解量子引力中的可观测性，我们必须结合广义相对论和量子理论中对可观测性的现有认知。从量子理论中我们得知，该理论能预测的是: 海森堡分界一侧，在可视为经典的环境中，引力场某一性质在相互作用中实现的概率。让这一框架成立的自然简洁关键，就是将海森堡分界等同于四维时空区域 $\mathcal{R}$ 的边界 [37]。不难发现，相对论引力中的所有实际观测都可以表示为这种形式。

It is particularly convenient to take $\mathcal{R}$ to be compact and bounded by a 3 d surface $\sum$ formed by the union of a past and a future spacelike surfaces $\sum_-$ and $\sum_+$ joined along a two sphere. The theory is naturally expressed in the time gauge on these surfaces. This setting permits us to interpret the LQG transition amplitudes as transition amplitudes from $\sum_-$ to $\sum_+$. Quantum states on $\sum_-$ and $\sum_+$ represent quantum geometries on these surfaces. These are interpreted as interactions between the gravitational field on $\mathcal{R}$ and the rest of the universe, across the Heisenberg cut defined by $\sum$.

将 $\mathcal{R}$ 取为紧致区域、并由一张 3 d 曲面 $\sum$ 包围是尤其方便的；$\sum$ 由过去类空曲面 $\sum_-$ 和未来类空曲面 $\sum_+$ 沿二维球面连接而成。该理论自然可以在这些曲面的时间规范下表述。这个设置允许我们将圈量子引力的跃迁振幅诠释为从 $\sum_-$ 到 $\sum_+$ 的跃迁振幅。$\sum_-$ 和 $\sum_+$ 上的量子态描述这两个曲面的量子几何。它们被诠释为 $\mathcal{R}$ 内的引力场与宇宙其余部分，通过 $\sum$ 定义的海森堡分界发生的相互作用。

This setting works very well in the two contexts where we expect quantum gravitational phenomena to be non-negligible: early cosmology and around black hole singularities. (See below.)

该设置在我们预期量子引力效应不可忽略的两个场景中都非常适用: 早期宇宙学和黑洞奇点附近。(见下文。)

A preparation and (a complete) measurement at the Heisenberg cut, namely, at $\sum$, determines the eigenvalues of a complete set of (partial) observables and a corresponding eigenstate $|\Psi\rangle \in \mathcal{H}_b$. (Semiclassical coherent stares are more convenient in some applications.) The dynamics is then given by a single bra $\langle W|$ on $\mathcal{H}_b$. This is the covariant version of the transition amplitude

在海森堡分界 (即 $\sum$ 处) 的制备和 (完整) 测量，会确定一组完备 (偏) 可观测量的本征值，以及对应的本征态 $|\Psi\rangle \in \mathcal{H}_b$ 。(半经典相干态在部分应用中更为方便。) 动力学则由 $\mathcal{H}_b$ 上的单个左矢 $\langle W|$ 给出，这就是跃迁振幅的协变形式

$$\langle W \mid \psi\rangle = \langle \Psi_+ |P| \Psi_- \rangle \tag{13}$$

where P is discussed in section "General Relativistic Evolution," the boundary Hilbert space is discussed in the last section, and $|\Psi_\pm\rangle$ are quantum states of the geometry of, respectively, $\sum_\pm$.

其中 P 在「广义相对论演化」一节讨论，边界希尔伯特空间在上一节讨论，$|\Psi_\pm\rangle$ 分别是 $\sum_\pm$ 几何的量子态。

Formally, if the measured quantities correspond to the 3 d (intrinsic) geometry g on $\sum$ and we call the corresponding eigenstate Ψ_g we may write

形式上，若测量量对应 $\sum$ 上的 3 d (内禀) 几何 g ，我们将对应本征态记为 Ψ_g ，可以写作

$$\langle W \mid \Psi_g \rangle = \int_{\partial g_4 = g} D g_4 e^{-\frac{i}{\hbar}\int \sqrt{-g_4} R[g_4]}, \tag{14}$$

where the (ill defined) functional integration of the exponent of the Einstein-Hilbert action is over the 4 d geometries g_4 on $\mathcal{R}$ bounded by g on $\sum$, as originally suggested by John Wheeler and Charles Misner [38]. The spinfoam formalism can be viewed as a way to transform the last formula into something well defined and computable, within arbitrary truncations.

其中对爱因斯坦-希尔伯特作用量指数的 (原本定义不完善) 泛函积分，是对 $\sum$ 上 g 包围的 $\mathcal{R}$ 区域内的 4 d 几何 g_4 进行的，这一思路最初由约翰·惠勒和查尔斯·米斯纳提出 [38]。自旋泡沫形式论可以看作是在任意截断下，将上述公式转化为定义明确、可计算形式的方法。

In a semiclassical regime we expect the classical dynamics of general relativity to be recovered from the approximation

在半经典区域，我们预期可以通过近似得到广义相对论的经典动力学:

$$\langle W \mid \Psi_g \rangle \sim \sum_n e^{-\frac{i}{\hbar} S_n[g]}, \tag{15}$$

where the sum is over the different solutions $g_4[g]$ of Einstein equations on $\mathcal{R}$ that induce the 3-metric g on $\sum$ and

其中求和遍历爱因斯坦方程在 $\mathcal{R}$ 上满足边界条件的所有不同解 $g_4[g]$，这些解在 $\sum$ 上诱导出 3-度量 g，且

$$S_n[g] = \int_{\mathcal{R}} \sqrt{-g_4[g]} R[g_4[g]] \tag{16}$$

is the corresponding Hamilton function of general relativity [1], namely, the value of the action on a solution with given boundary data. As well known, full knowledge of the Hamilton function is essentially equivalent to knowledge of the solution of the equations of motion. (To see how this still works in the generally covariant case, see [1].)

是广义相对论对应的哈密顿函数 [1]，即给定边界数据下，作用量在解上的取值。众所周知，对哈密顿函数的完整认知本质上等价于掌握运动方程的解。(关于它在广义协变情形下仍然成立的说明，参见 [1]。)

From the conceptual point of view that concerns us here, we notice that the observables of quantum gravity can be chosen to be sitting on the Heisenberg cut $\sum$ and be partial observables. Importantly, they do not need to be fully gauge invariant, because they represent interactions between the quantum system studied and ("the measurement apparatus on") the boundary of the classical context. What the theory provides, then, are transition amplitudes between partial observables, and these give the physical predictions, as illustrated above.

从我们当下关注的概念视角来看，我们注意到量子引力的可观测量可以被设定为位于海森堡分界 $\sum$ 上，属于部分可观测量。重要的是，它们不需要是完全规范不变的，因为它们描述的是被研究的量子系统和经典环境边界 (「测量仪器所在处」) 之间的相互作用。因此如上文所述，该理论给出的是部分可观测量之间的跃迁振幅，而这些跃迁振幅就是物理预测。

An alternative way to derive gauge-invariant predictions from quantum gravity is to write fully gauge-invariant observables on the phase space of the theory. If we had a sufficiently rich family of such gauge-invariant observables, these could be used to compute transition amplitudes directly, because fully gauge-invariant observables in general relativity are like operators in the Heisenberg picture. The scalar product between their eigenstates would directly give transition amplitudes, exhausting all relevant dynamical information. Many authors have explored this path [39-56]. This strategy involves an infamously hard task, as discussed in detail by many of the authors cited, from a variety of points of view: Dirac observables are highly nonlocal and obey a nonlocal algebra in a general relativistic setting. The difficulty has been addressed in various manners, such as expanding around flat space, or coupling suitable matter fields to use as references, or using nonlocal dressings of fields, or using geodesics from infinity to define bulk localization, or considering the particular class of Dirac observables called the ”evolving constants of the motion” [57]. Here we shall not review those intriguing possibilities, for which we refer to the authors cited.

从量子引力推导规范不变预言的另一种方法，是在该理论的相空间上写出完全规范不变的可观测量。如果我们拥有足够丰富的这类规范不变可观测量，就可以用它们直接计算跃迁振幅，因为广义相对论中的完全规范不变可观测量就类似于海森堡绘景下的算符。其本征态之间的标量积可以直接给出跃迁振幅，囊括所有相关动力学信息。已有许多研究者探索了这一方向 [39-56]。正如多位引文中的作者从不同角度详细讨论过的，该策略面临一项众所周知的难题: 在广义相对论框架中，狄拉克可观测量具有高度非局域性，并且服从非局域代数。研究者已经通过多种方式处理这一难点，例如在平直空间附近展开，耦合合适的物质场作为参考，使用场的非局域修饰，利用从无穷远出发的测地线定义体域局域化，或是研究被称为“运动演化常数”的特定狄拉克可观测量类别 [57]。本文我们不会回顾这些有意思的方向，相关内容请参考上述引文中的作者。

The boundary strategy for computing predictions in quantum gravity described above, instead, circumvents the infamous difficulty of writing fully gauge-invariant observables (or ”Dirac observable”) in general relativity as explicit functions of the theory’s phase space. Explicit knowledge of these functions is not necessary to extract information from a quantum theory of gravity. In fact, it is not necessary to extract information from the classical theory of gravity either. This is evident from the fact that we do not know how to write such quantities on the phase space of general relativity, and yet in general relativistic physics, astrophysics, and cosmology, we can routinely do general relativistic observations, measurements, and predictions, as illustrated in section ”Observability in Gravitational Physics.” Clearly there are ways of extracting physically meaningful information from a general relativistic theory even without being capable of writing its Dirac observables.

反之，前文介绍的量子引力预言计算边界方案规避了在广义相对论中将完全规范不变可观测量 (或“狄拉克可观测量”) 写为理论相空间显式函数这一著名难题。要从量子引力理论中提取信息，并不需要掌握这些函数的显式形式；实际上，即便从经典引力理论中提取信息，这也不是必需的。这一点很明显: 我们至今不知道如何在广义相对论的相空间上写出这类量，但广义相对论物理、天体物理和宇宙学中，我们依然可以常规地完成广义相对论观测、测量和预言，正如“引力物理中的可观测量”一节所举示例所示。显然，即使无法写出狄拉克可观测量，我们依然有办法从广义相对论理论中提取有物理意义的信息。

In other words, to make predictions about the behavior of a covariant system, it is not necessary to know explicit its physical Hilbert space and the operators which leave the physical Hilbert space invariant. Knowledge of the extended (or kinematical) Hilbert space $\mathcal{H}_{\rm ex}$ and the transition amplitudes is sufficient. For example, consider a particle in two dimensions described in a covariant manner with states $|x,t\rangle \in \mathcal{H}_{\rm ex}$. The full theory can be expressed in terms of the transition amplitudes

换句话说，要对协变系统的行为做出预言，并不需要显式知道系统的物理希尔伯特空间，以及保持物理希尔伯特空间不变的算符。只要知道扩展 (即运动学) 希尔伯特空间 $\mathcal{H}_{\mathrm{ex}}$ 和跃迁振幅就足够了。例如，考虑一个用协变方式描述、以 $|x,t\rangle \in \mathcal{H}_{\mathrm{ex}}$ 为态的二维粒子，整个理论可以用跃迁振幅表示为

$$W(x,t;x',t') \equiv \langle x,t\,|P|\,x',t'\rangle, \tag{17}$$

where P is an operator on $\mathcal{H}_{\mathrm{ex}}$, without reference to a physical Hilbert space of solutions of the "Wheeler-DeWitt equation" whose solutions define the physical Hilbert space. If we can deparametrize the theory, the explicit form of the transition functions can be written in the well-known form

其中 P 是 $\mathcal{H}_{\mathrm{ex}}$ 上的算符，无需参考由“惠勒-德维特方程”的解定义的物理希尔伯特空间。如果我们可以对理论去参数化，跃迁函数的显式形式可以写成大家熟悉的形式:

$$W(x,t;x',t') \equiv \langle x\left|e^{-\frac{i}{\hbar}H(t-t')}\right|x'\rangle, \tag{18}$$

but this expression may be ill defined in the general case, while (17) remains well defined. Intuitively, and in the cases where these equations are well defined,

但这个表达式在一般情况下可能不是良定义的，而 (17) 仍然保持良定义。直观来看，在这些方程良定义的情形中，

$$W(x,t;x',t') \sim \langle x,t\,|\delta(C)|\,x',t'\rangle \sim \int_{(x',t')\to(x,t)} DX e^{\frac{i}{\hbar}S[X]} \tag{19}$$

where C is the Hamiltonian constraint, S is the action, and the integral is a Feynman integral over paths X . In the covariant formulation of LQG, the (truncated, see below) quantities W are defined by the spinfoam amplitude and are functions of boundary states representing 3-geometries, as in (14). There is no need to "deparametrize" or finding Dirac observables, to study quantum gravitational processes. Quantum transition amplitudes, like predictions in classical general relativity, can be formulated and computed using only partial observables, working with gauge-dependent quantities and exploiting the third of the three interpretations of general covariance listed in section "Observability in Gravitational Physics."

其中 C 是哈密顿约束，S 是作用量，积分是路径 X 上的费曼积分。在圈量子引力 (LQG) 的协变表述中，(经截断的，见下文) 量 W 由自旋泡沫振幅定义，是表示 3 几何的边界态的函数，正如 (14) 式所示。研究量子引力过程不需要“去参数化”或者寻找狄拉克可观测量。和经典广义相对论中的预言一样，量子跃迁振幅可以仅利用部分可观测量、使用规范依赖量，并依托“引力物理中的可观测量”一节列出的三种广义协变诠释中的第三种来构造和计算。

The relational interpretation of quantum theory [58,59] is a natural setting when quantum gravity is formulated in this manner. The relational structures of space and time merge naturally and beautifully with the relational structure of quantum theory: the Heisenberg cut is identified with spacetime partitions. Notice that in the Copenhagen version, we still need an outside classical spacetime. Not so in the relational interpretation, where what matters is only that there is am (relational) boundary between two systems, without any presupposed geometry on this boundary.

量子理论的关系诠释 [58,59] 是以此种方式构造量子引力的自然框架。空间和时间的关系结构，与量子理论的关系结构自然优美地融合: 海森堡 cut 对应于时空分割。注意在哥本哈根版本中，我们仍然需要一个外部经典时空，而关系诠释并非如此，在关系诠释中，关键仅在于两个系统之间存在一个 (关系) 边界，不需要这个边界上有预先假定的几何。

However, the problem of quantum gravity and the problem of the interpretation of quantum mechanics are distinct and to a large extent independent.

但量子引力问题和量子力学诠释问题是两个不同的问题，在很大程度上相互独立。

A concrete example of utilization of LQG transition amplitudes is given by the calculations of what happens at the end of the evaporation of a black hole. Most of spacetime can be treated classically, because quantum gravitational effects are negligible. Not so the high curvature region surrounding the classical (unphysical, because of quantum gravity) singularity and the horizon near the end of the evaporation. This is the compact quantum region $\mathcal{R}$. A 3d surface surrounding it can be chosen and the LQG quantum transition amplitudes describing what can happen at the end of the evaporation can be explicitly studied: see [60-64]. (This is possible because a classical exact solution for the exterior exists [65].)

LQG 跃迁振幅的一个具体应用实例，是对黑洞蒸发终点的物理过程计算。大部分时空可做经典处理，因为量子引力效应可忽略；但经典奇点 (因量子引力的存在，该奇点并不物理) 周围的高曲率区域，以及蒸发末期附近的视界则并非如此。这就是有紧致边界的量子区域 $\mathcal{R}$ 。我们可以选取一个包围该区域的三维曲面，明确研究描述蒸发终点可能发生过程的 LQG 量子跃迁振幅: 参见文献 [60-64]。(这之所以可行，是因为外部存在精确经典解 [65]。)

Another concrete example is the use of this covariant formalism to study the big bang [66,67]. In this case the surface $\sum$ can be taken to be a single spacelike surface with the topology of a 3-sphere after the big bang, to describe the transition from nothing to a cosmological space (as in the Hartle-Hawking scenario), or, alternatively, as two disconnected spacelike surfaces with the topology of a 3-sphere, to describe a big bounce, as in loop quantum cosmology [68]. In this cosmological context, the average value of the spins can be taken as the independent variable (a discretized version of the cosmological scale factor), in terms of which the dynamics of the fluctuations of the rest of the geometry evolves.

另一个具体实例是利用该协变形式论研究大爆炸 [66,67]。在这种情况下，可将曲面 $\sum$ 取为大爆炸后一个具有三维球面拓扑的类空单曲面，用来描述从无到有生成宇宙空间的跃迁 (如同哈特-霍金场景)；或者也可取为两个不连通的三维球面拓扑类空曲面，用来描述圈量子宇宙学中的大反弹过程 [68]。在这个宇宙学背景下，可将自旋的平均值作为自变量 (它是宇宙学标度因子的离散版本)，其余几何涨落的动力学都可依此自变量演化。

Truncation, Finite Graphs, and Finite Spinfoams

截断、有限图与有限自旋泡沫

The bra $\langle W|$ that gives the dynamics is defined in covariant LQG in terms of spinfoam amplitudes, order by order in a suitable sequence of truncations that represent increasingly fine approximations. See [2] for a detailed technical introduction of these. Here we only discuss the conceptual structure of the theory.

给出动力学的辫子 $\langle W|$ 在协变圈量子引力中由自旋泡沫振幅定义，按合适的截断序列逐阶展开，这些截断代表精度逐步提升的近似。相关详细技术介绍见文献 [2]，本文仅讨论该理论的概念结构。

A general remark is important here, to dispel a recurring conceptual confusion: the idea that a quantum theory, and in particular a quantum theory of gravity, could only describe elementary components of nature. There is no reason to expect so, and this is not the right way of viewing and using quantum theory. Quantum theory is not a theory about the elementary components of reality: it is a theory about the quantum behavior of any physical variable, irrespectively on whether this is elementary or composite. The angular momentum of a molecule, for instance, is quantized and can be used in computing transition amplitudes in the dynamics of the molecule, independently from the internal quarks's structure of the molecule.

此处有一条重要的一般性说明，用以消除一个反复出现的概念混淆: 即认为量子理论，尤其是量子引力理论，只能描述自然的基本组分。没有任何理由支撑这一观点，这也不是看待和使用量子理论的正确方式。量子理论并非关于实在基本组分的理论: 它是关于任意物理变量量子行为的理论，无论该变量是基本的还是复合的。例如，分子的角动量是量子化的，它可用于计算分子动力学中的跃迁振幅，和分子内部夸克结构无关。

In the same manner, there is no reason to see the quantization of some aspects of the metric field as a description of elementary components: any variable, at any scale, behaves quantum mechanically and LQG is the description of quantum properties of gravitational degrees of freedom, at any relevant scale. A state does not represent a "thing": it represents the outcome of an interaction in which certain observables take on certain values. No measurement delivers an infinite amount of information. Any measurement captures a finite number of degrees of freedom only. An effective theoretical description of a given phenomenon needs only to refer to the degrees of freedom that are relevant for that phenomenon.

同理，没有理由将度量场某些方面的量子化视为对基本组分的描述: 任意尺度下的任意变量都遵循量子力学行为，而圈量子引力描述任意相关尺度下引力自由度的量子性质。量子态不代表某个“物体”: 它代表某一相互作用中可观测量取特定值的结果。没有测量能传递无限多信息，任何测量都仅能捕获有限个自由度。对特定现象的有效理论描述只需要涉及该现象相关的自由度。

This is also true in quantum field theory. In standard quantum field theory calculations are always performed on finite lattices and finite graphs. For instance, in lattice QCD hadron's masses are computed on lattices of finite size that are large enough to include the hadron and fine enough to see the quarks' wavelengths, but no more. Analogously, collisions in QED and in the electroweak theory are computed order by order in a perturbation expansion: at each order there are a maximum number of (real and virtual) particles involved and therefore, again, only a finite number of degrees of freedom involved. For this reason, both the lattices concretely used in QFT calculations and the (Feynman) graphs in QFT perturbation theory are finite (i.e., they have a finite number of vertices).

这一点在量子场论中同样成立。标准量子场论的计算始终在有限格点和有限图上进行。例如格点量子色动力学中，强子质量是在有限尺寸格点上计算得到的: 这类格点尺寸大到足以容纳强子，精度高到足以观测夸克波长，仅此而已。类似地，量子电动力学和电弱理论中的碰撞过程按微扰展开逐阶计算: 每一阶都仅涉及有限数量的实粒子和虚粒子，因此同样仅涉及有限个自由度。正因如此，量子场论计算中实际使用的格点和微扰论中的费曼图都是有限的 (即顶点数量有限)。

The same holds in loop quantum gravity: concrete calculations involve spinnet-works and spinfoams with finite graphs and finite two complexes. The calculation is expected to provide results that approximate the physical behavior of phenomena where the corresponding degrees of freedom play a role. When describing a phenomenon (like the end of the evaporation of a black hole or the possible bounce of the primordial universe), we have to single out the degrees of freedom that may play a relevant role in the corresponding dynamics and describe the process in terms of these, not in terms of everything [69, 70].

这一点在圈量子引力中同样成立: 具体计算都涉及带有限图和有限二维复形的自旋网络与自旋泡沫，人们预期这类计算得到的结果可以近似对应自由度相关现象的物理行为。当描述某一现象时 (比如黑洞蒸发的末态或是原初宇宙可能的反弹)，我们只需要筛选出在对应动力学中发挥相关作用的自由度，用这些自由度描述过程即可，无需涉及所有事物 [69, 70]。

Suggestions that calculations on finite graphs and finite spinfoams are unreliable are therefore conceptually ill-founded.

因此，认为有限图和有限自旋泡沫上的计算不可靠的观点，在概念上是毫无根据的。

A measurement of the geometry that captures a finite number of degrees of freedom can be modeled as follows. Given a 3d metric space with geometry g , consider a simplicial decomposition of the space and call A_ℓ the areas of the 2- simplices ℓ , and $\mathbf{n}_\ell \cdot \mathbf{n}_{\ell'}$ the angle between two vectors normal to two 2-simplices ℓ , and ℓ' bounding the same 3-simplex n , in an arbitrary point of ℓ and ℓ' , parallel transported to an arbitrary internal point of n . Then Equation (1) defines a family of quantities that measures the geometry g at some scale. These quantities do not commute in quantum theory. A (smaller) set of commuting quantities is given by the areas A_ℓ of the two simplices and the volumes v_n of the three simplices. A volume operator V_n is defined on $\mathcal{H}_\Gamma$, where Γ is the graph dual to the cellular decomposition. The operators (A_ℓ, V_n) form a commuting set of operators in $\mathcal{H}_\Gamma$ which is maximal up to some signs that we disregard here for simplicity. These operators have discrete spectrum [15]. Let $|\Gamma, j_\ell, v_n\rangle$ be a basis in $\mathcal{H}_\Gamma$ that diagonalizes this set. The states $|\Gamma, j_\ell, v_n\rangle$ can be interpreted to represent the outcomes (j_ℓ, v_n) of these measurements, in the standard sense of quantum theory.

捕获有限个自由度的几何测量可按如下方式建模。给定具有几何结构 g 的三维度量空间，对该空间做单纯分解，记 A_ℓ 为二维单纯形 ℓ 的面积，记 $\mathbf{n}_\ell \cdot \mathbf{n}_{\ell'}$ 为两张二维单纯形 ℓ 的法向量之间的夹角，ℓ' 界定同一个三维单纯形 n ，取 ℓ 和 ℓ' 中任意一点，将法向量平行移动到 n 的任意内点计算该夹角。而后方程 (1) 定义了一族在某一尺度下测量几何 g 的物理量。这些物理量在量子理论中不对易。由二维单纯形的面积 A_ℓ 和三维单纯形的体积 v_n 构成的 (更小的) 集合是对易物理量集合。体积算符 V_n 定义在 $\mathcal{H}_\Gamma$ 上，其中 Γ 是对偶于胞腔分解的图。算符 (A_ℓ, V_n) 在 $\mathcal{H}_\Gamma$ 上构成一个极大对易算符集合，为简便起见我们此处忽略符号问题。这些算符具有离散谱 [15]。令 $|\Gamma, j_\ell, v_n\rangle$ 为 $\mathcal{H}_\Gamma$ 中该集合对角化的一组基。在量子理论的标准意义下，态 $|\Gamma, j_\ell, v_n\rangle$ 可被解释为代表这些测量的结果 (j_ℓ, v_n) 。

This does not mean that a state like $|\Gamma, j_\ell, v_n\rangle$ gives a complete description of reality in a certain spacetime region. It only refers to a subset of degrees of freedom measured. The theory that describes these degrees of freedom is a good description of reality to the extent the dynamics of these degrees of freedom is not too affected by others.

这并不意味着像 $|\Gamma, j_\ell, v_n\rangle$ 这样的态就能完整描述某一时空区域内的实在。它仅对应于被测的那部分自由度。只要这部分自由度的动力学受其余自由度的影响不大，描述这些自由度的理论就是对实在的良好描述。

In the covariant formulation, the LQG transition amplitudes are defined in terms of a sequence of 4 d truncations, after fixing a relevant family of boundary states (say in $\mathcal{H}_\Gamma$). Each truncation is defined by the choice of a 2-complex $\mathcal{C}$ having Γ as boundary. The spin foam amplitudes define a bra $\langle W_{\mathcal{C}}|$ on $\mathcal{R}_\Gamma$ [2, 71 − 73] . The theory is well defined if refining the 2-complex the amplitude converges. Numerical calculations give some partial positive indications that this can be the case that exists; see for instance [74].

在协变表述中，固定一族相关边界态后 (例如在 $\mathcal{H}_\Gamma$ 中)，LQG 跃迁振幅由一系列 4 d 截尾定义。每个截尾由选择一个以 Γ 为边界的 2 复形 $\mathcal{C}$ 定义。自旋泡沫振幅在 $\mathcal{R}_\Gamma$ [2, 71 − 73] 上定义了一个左矢 $\langle W_{\mathcal{C}}|$ 。当细化 2 复形时振幅收敛，该理论就是良定义的。数值计算给出了部分有利迹象表明这种情况确实存在；参见例如文献 [74]。

Physical Discreteness

物理离散性

It is important not to confuse the discreteness introduced by the various truncations used to define the theory (the graph of the spinnetworks, the two complexes of the spinfoams) with the physical Planck scale discreteness predicted by LQG. The first is only a theoretical tool, analog to the lattice of lattice QCD. The second (absent in QCD) is a hard physical prediction of the theory and the most characteristic feature of LQG. It is the analog of the discreteness of the spectra of the energy of the hydrogen atom or non-relativistic harmonic oscillator or the discreteness of photons. It is derived in the theory from the spectral analysis of the operators describing the geometry [15]. It is compatible with the local Lorentz invariance of the theory [75]. It is this physical discreteness which is responsible for the ultraviolet finiteness of LQG and for the resolution of the singularities of general relativity [76,77].

必须注意不要将定义理论时各类截断引入的离散性 (自旋网络的图、自旋泡沫的二维复形) 与 LQG 预测的普朗克尺度物理离散性混为一谈。前者只是理论工具，类似于量子色动力学 (QCD) 中的晶格。后者 (QCD 中不存在这类离散性) 是该理论坚实的物理预言，也是 LQG 最具标志性的特征。它类比于氢原子或非相对论谐振子的能谱离散性，或是光子的离散性。它是由理论中描述几何的算符做谱分析推导得出的 [15]，且与理论的局域洛伦兹不变性相容 [75]。正是这种物理离散性保证了 LQG 的紫外有限性，解决了广义相对论的奇点问题 [76,77]。

The expression of the transition amplitudes as a spinfoam sum has much in common with a standard lattice discretization of a Feynman sum over histories [78], like the one that defines lattice QCD. However, there is a crucial difference [79]: in theories like lattice QCD the full quantum theory is recovered by sending the number of lattice sites to infinity as well as the lattice spacing to zero. Because of the underpinning diffeomorphism invariance, only the first of these limits (i.e., refining the two complexes) is required in LQG. See a detailed discussion in [79].

将跃迁振幅表示为自旋泡沫求和，这与费曼历史求和的标准晶格离散化方法 [78](比如定义晶格 QCD 的方法) 有诸多共性。但二者存在关键差异 [79]: 在晶格 QCD 这类理论中，当晶格点数趋于无穷、晶格间距趋于零时，才能得到完整量子理论。而在 LQG 中，由于基础微分同胚不变性，仅需要取第一个极限 (即细化二维复形)，详细讨论见 [79]。

Here ”limit” must be understood in the sense of potential, not actual. What we do in physics is to compute transition amplitudes within approximations. This is always done within a finite truncation, as we do in standard perturbative QFT and lattice QCD. Therefore the theory can be formally defined by the continuous limit, but the actual theory to be used is always at arbitrary but finite truncation (This observation may either be seen as a simple pragmatic consideration or, perhaps, as a way to question the physical relevance of the actual limit theory [80].).

此处的“极限”必须理解为潜在极限，而非实际取到的极限。我们在物理研究中都是在近似下计算跃迁振幅，这类计算始终是在有限截断内完成的，就像我们在标准微扰量子场论和晶格 QCD 中所做的那样。因此理论可以在形式上由连续极限定义，但实际使用的理论始终是任意但有限截断下的理论 (这一观察既可以被看作简单的实用考量，也或许可以用来质疑实际极限理论的物理关联性 [80]。)。

A different question is how we recover classical general relativity. For this, we have to take both the continuum limit and a ”classical” limit: namely, look at scales large with respect to the Planck scale. This is usually implemented as a large spin limit. For a while, the LQG literature contained the wrong expectation (giving rise to an apparent ”flatness problem”) that the classical limit could be taken before, and independently from, the continuum limit. This is not the case: the two limits must be taken together; see [2, 81-84] (Specifically, at fixed triangulation, the LQG amplitudes approximate sufficiently well the dynamics of discretized general relativity (Regge theory) only if the triangulation is sufficiently fine.).

如何还原经典广义相对论是另一个问题。为此我们必须同时取连续极限和“经典”极限: 也就是研究远大于普朗克尺度的尺度。这通常实现为大自旋极限。有一段时间，LQG 文献中存在错误的预期 (引发了 apparent“平坦性问题”)，认为经典极限可以在连续极限之前、独立于连续极限取出。事实并非如此: 两个极限必须同时取；见 [2, 81-84](具体而言，在固定三角剖分下，只有当三角剖分足够精细时，LQG 振幅才能足够好地近似离散广义相对论 (里奇理论) 的动力学。)。

Three Distinct Notions of Time

三种不同的时间概念

In moving from non-relativistic physics to quantum gravity, the notion of time undergoes alterations similar to the notion of space. However, the notion of time is more subtle than the notion of space, raising further issues.

从非相对论物理向量子引力推进的过程中，时间概念发生的变化与空间概念类似。但时间概念比空间概念更微妙，还引出了更多问题。

In the case of space, we observed that clarity is obtained by distinguishing the common relational notion of space, according to which objects are spatially located with respect to one another - a notion still in play in quantum gravity - from the Newtonian notion of space, as a continuous metric manifold with an Euclidean geometry, which emerges only in approximations.

就空间而言，我们已经发现，区分两种概念即可获得清晰性: 一种是通用的关系空间概念——根据这一概念，物体相对于彼此处于空间位置，这一概念在量子引力中依然适用；另一种是牛顿的空间概念，即具有欧几里得几何的连续度量流形，这一概念仅在近似中成立。

In the case of time, the same distinction holds. In everyday life we use a relational notion of time. Time is just a counting of happenings in successions: for example, the succession of days and years. It is a fact of nature that there are such successions of events. Newtonian physics, on the other hand, postulates the existence of a physical time that is independent from any succession of events and has a rich structure: it has a metric structure; it is the same all over the universe, defining a global simultaneity; and so on. As well known, many features of such Newtonian time are approximations: they do not describe correctly the actual temporal structure of reality. There is no single canonical clock variable in the universe, and no global simultaneity, except in dramatic approximations like homogeneity and isotropy in cosmology.

就时间而言，同样的区分也成立。在日常生活中我们使用关系时间概念。时间只是对接连发生的事件的计数: 例如昼夜与年份的接续。自然界中本就存在这样的事件接续。另一方面，牛顿物理学假定存在一种独立于所有事件接续的物理时间，且该时间具有丰富的结构: 它具备度量结构；在全宇宙都是统一的，能定义全局同时性；诸如此类。众所周知，牛顿时间的许多特性都是近似: 它们无法正确描述现实真实的时间结构。宇宙中不存在单一的标准时钟变量，也不存在全局同时性，只有在宇宙学均匀性与各向同性这类强近似下才成立。

The absence of a single preferred time variable and the fact that no single variable has all the features typical of Newtonian time is just a fact of nature and is the reason for the generalization of mechanics illustrated in section ”General Relativistic Evolution.” The formal structure illustrated in that section permits to define both the classical and the quantum dynamics coherently without having to specify a preferred time variable. As observed, non-relativistic physics describes evolution as change of the variables in time, and relativistic physics describes evolution as change of the variables with respect to one another.

不存在单一的优先时间变量，也没有任何变量具备牛顿时间的全部典型特征，这本就是自然的事实，这也是“广义相对论演化”一节所阐释的力学推广的原因。该节所阐释的形式结构允许我们在不指定优先时间变量的情况下，连贯地定义经典动力学和量子动力学。正如前文所说，非相对论物理学将演化描述为变量随时间的变化，而相对论物理学将演化描述为变量相对于彼此的变化。

In the literature, a big deal had been made about the alleged existence of a ”problem of time” due to this absence of a canonical time. The confusion in this issue stems from a mixing up two distinct questions:

在文献中，由于不存在标准时间，所谓存在“时间问题”的说法被炒得沸沸扬扬。这个问题的混乱源于混淆了两个不同的问题:

1. The first question is to understand how to describe dynamical evolution in a relativistic setting when there is no canonical time variable.

> 1. 第一个问题是理解当不存在标准时间变量时，如何在相对论框架下描述动力学演化。

2. The second problem is raised by our strong feeling that time ”flows” in a sense that makes it different from any other physical variable. In a Newtonian theory, we identify the flow of time with the change in the canonical variable T of the Newtonian formalism. But we cannot do so in a theory formulated in a way that does not select any time variable as special.

> 2. 第二个问题来自我们的强烈感受: 时间“流动”，这一点让它不同于其他所有物理变量。在牛顿理论中，我们将时间的流动等同于牛顿形式体系中标准变量 T 的变化。但在不指定任何特殊时间变量的理论中，我们无法这么做。

The first of these two problems can be solved classically, as illustrated in section ”General Relativistic Evolution”: a dynamical theory does not require a specific time variable to be defined. All predictions by a general relativistic theory can be obtained without specifying a canonical, or ”special,” time variable. So, this question can be consistently answered. The quantities predicted by the theory are values of some variables when other variables have given values.

> 上述两个问题中的第一个可以在经典层面解决，正如“广义相对论演化”一节所阐释的: 动力学理论不需要预先定义特定的时间变量。广义相对论理论的所有预言都可以在不指定标准或“特殊”时间变量的情况下得到。因此这个问题可以得到一致的解答。该理论预言的物理量，就是当其他变量取给定值时，某些变量的取值。

A corresponding quantum formalism can also be defined, as shown in section ”General Relativistic Evolution.” Alleged ”quantum” solutions of this same problem, such as the Page-Wootters construction [41], are nothing else than this same solution expressed in the quantum domain.

> 相应的量子形式体系也可以被定义，正如“广义相对论演化”一节所示。针对同一问题的所谓“量子”解，例如佩奇-伍特斯构造 [41]，不过是该解在量子领域的表述而已。

The second of the above problems, on the other hand, is rooted on a conceptual misunderstanding. We do experience a flow of time, of course. To understand this experience we should look at our experience as it is in reality, and not assume that our experience reaches out directly to the deep and general structure of reality.

> 另一方面，上述第二个问题植根于概念误解。当然，我们确实会经验到时间的流动。要理解这种经验，我们应当从经验的实际样貌出发，而非假定我们的经验能直接触及实在深层的普遍结构。

What we experience is due to the specific and complex situation in which we are. Not only our experience is in the Newtonian limit (so that we misinterpret aspects of this limit for universal features of nature), but it is also strongly marked by the fact that we access a small subset of the degrees of freedom of nature; hence, we experience macroscopic coarse grained variables that happen to have thermodynamic properties. In particular, we happen to live in a universe with a strong entropy gradient, where the behavior of these macroscopic observables has a marked irreversible character. (We do not know why. A hypothesis for the reason of this is in [85], but this is irrelevant here.) From the perspective of the fundamental theory, this fact is accidental.

我们的经验源于我们所处的特殊复杂情境。我们的经验不仅处于牛顿极限 (因此我们会将该极限的某些方面误认作自然的普遍特征), 还被一个事实深刻影响: 我们只能接触到自然自由度的一小部分; 因此，我们经验到的宏观粗粒变量恰好具有热力学性质。尤为特殊的一点是，我们恰好生活在一个存在强熵梯度的宇宙中，这些宏观可观测量的行为在此有着明显的不可逆性。(我们不知道这是为什么。对此的一种假说参见 [85]，但这和此处讨论无关。) 从基础理论的视角来看，这个事实是偶然的。

A direct consequence of this fact is that our local present has abundant traces of the past [86], and past low entropy allows macroscopic histories to branch [87, 88]. These facts determine the epistemic and the agential arrows of time, both aligned with the entropy gradient. These phenomena, not any preferred fundamental temporal variable, are principally responsible for the phenomenology of our experiential time. For an ample discussion of all this, see [89]. All this is very interesting, of course, but has nothing to do with quantum gravity.

这一事实的直接推论是，我们的局域当下存在大量过去的痕迹 [86]，而过去的低熵允许宏观历史发生分支 [87, 88]。这些事实决定了时间的认知箭头与作用箭头，二者都与熵梯度方向一致。这些现象，而非任何优先的基本时间变量，才是我们经验时间现象的核心成因。关于所有这些内容的详细讨论参见 [89]。当然，这一切都非常有意思，但和量子引力毫无关系。

(Incidentally, a consistent thermodynamic and statistical theory of the classical gravitational field is still missing, let alone for the quantum one. This is one of the reasons of the confusion surrounding issues like black hole entropy. For hints and clumsy attempts in this directions, see for instance [90-92].)

(顺带一提，经典引力场始终缺乏自洽的热力学与统计理论，更不用说量子引力场了。这是围绕黑洞熵这类问题存在诸多混淆的原因之一。关于这一方向的思路和不成熟尝试，参见例如 [90-92]。)

The notions of time that need to be distinguished in order to get clarity in quantum gravity are therefore three:

因此，要在量子引力研究中理清思路，需要区分三种不同的时间概念:

1. Relational time is the notion that allows us to say that two local events happen in a direct succession next to one another. This is the analog of relational space. This notion remains true in quantum gravity: we compute transition amplitudes for successions of local events.

1. 关系时间是让我们得以判断两个局域事件依次相邻发生的概念。它是关系空间的对应概念。这一概念在量子引力中仍然成立: 我们会计算局域事件序列的跃迁振幅。

2. Newtonian time is a quantity that is well defined only under numerous approximations taken. Various features of Newtonian time are lost one after the other as approximations are undone.

2. 牛顿时间是只有在做了大量近似之后才能良好定义的量。当这些近似被取消，牛顿时间的各项特征会逐一消失。

3. Experienced time includes a rich phenomenology that depends on the specific environment around us, especially the irreversibility due to the entropy gradient, and on the functioning of our brain and its functioning in terms of deliberations [93, 94].

> 3. 经验时间包含丰富的现象性，它依赖于我们周遭的特定环境，尤其是熵梯度带来的不可逆性，也依赖于我们大脑的运作及其在深思层面的功能 [93, 94]。

It is perhaps clarifying to distinguish a generic notion of ”change” from the specific concept of ”time.” By ”change” we may mean the most generic aspect of temporal contingency, in the following sense. We experience in the world that things can be in a certain way ”sometime” and different ”some other time.” This is a notion which is local (not global across the universe), not necessarily oriented, and does not require a single time variable to be described. That is, we describe the world in terms of a certain number of quantities: $(A, B, C, \ldots)$ and the functional dependencies between these - we can compile lists of observations $(A_1, B_1, C_1, \ldots) \ldots (A_2, B_2, C_2, \ldots) \ldots$, giving us the values of these quantities ”changed.” Physics gives us equations that constrain these changes. For instance, the change in an oscillator is described by the two partial observables (X, T) and their relation $f(X, T) = X - A\sin(\omega T - \phi) = 0$, where A and ϕ are constants. In this particular case, we can recognize ” T ” as what we usually call time, but in general relativity there is no such easy recognition, and in general none of the variables $(A, B, C, \ldots)$ have all the qualities we ascribe to time in Newtonian physics. By ”time” we indicate a particular variable among those describing the world that has a particular list of properties (for instance, it is monotonic along the change), and in the approximate description of the world obtained in the non-quantum, nonrelativistic limit is associated with the quantity measured by our clocks and with our experiential time: the sense of passing we have in our brain. So, change and time are different. The first is part of the conceptualization in quantum gravity, the second is not.

> 将泛化的「变化」概念和特殊的「时间」概念区分开或许有助于理清思路。我们用「变化」指代时间偶然性最普遍的特征，含义如下: 我们在世界中经验到，事物可以「某时」是某种状态，「另一时」又是另一种状态。这个概念是局域的 (而非覆盖全宇宙的全局概念)，不一定有方向性，也不需要用单一时间变量来描述。也就是说，我们用若干量来描述世界: $(A, B, C, \ldots)$ 以及这些量之间的函数依赖——我们可以整理观测列表 $(A_1, B_1, C_1, \ldots) \ldots (A_2, B_2, C_2, \ldots) \ldots$ ，得到这些量「变化后」的取值。物理学给出约束这些变化的方程。例如，振子的变化由两个部分可观测量 (X, T) 和它们的关系 $f(X, T) = X - A\sin(\omega T - \phi) = 0$ 描述，其中 A 和 ϕ 是常数。在这个特例中，我们可以认出「T」就是我们通常所说的时间，但在广义相对论中没法这么轻易认出来，而且一般来说，没有任何变量 $(A, B, C, \ldots)$ 具备我们在牛顿物理学中赋予时间的全部性质。我们用「时间」指代描述世界的诸变量中一个特殊变量，它具备一组特定性质 (例如，它随变化单调递增)，并且在非量子、非相对论极限下得到的近似世界描述中，它对应我们时钟测量的量，也对应我们的经验时间——即我们大脑中拥有的流逝感。因此，变化和时间是不同的。前者是量子引力概念框架的一部分，后者则不是。

Conclusion

结论

A large number of important technical issues are open in quantum gravity (For instance, the infrared ”bubble” divergences [95-98]), not to mention the persistent lack of direct empirical support. But LQG has at its disposal not only a powerful mathematical formalism that represents a tentative theory of gravity, but also a coherent conceptual picture within which a possible understanding of quantum spacetime can be framed.

量子引力领域仍有大量重要技术问题尚未解决 (例如红外“泡泡”发散 [95-98])，更不用说一直缺乏直接实证支持。但圈量子引力不仅拥有一套强有力的数学形式体系，构成了一个试探性引力理论，还具备一套连贯的概念框架，可以在此框架内构建对量子时空的可能理解。

The relational notions of space and time that are familiar from our common experience remain useful in quantum gravity: events can be ”next” to one another spatially and ”next” to one another temporally. Not so the structure of a general relativistic spacetime, which only emerges in approximations. A general covariant formalism for dynamics is well defined and clear: it is based on the notion of partial observables: quantities that can be measured but in general cannot be predicted by themselves. The dynamical theory gives the correlations between these, both in the classical and quantum domains. The observable quantities in quantum gravity are the same as those of general relativity: in principle, any measurement in relativistic gravitational physics is also a measurement in quantum gravity. (Any measurement in relativistic gravitational physics can be represented as performed across a 3d surface in the form described above).

我们日常经验中熟悉的关系性时空概念在量子引力中仍然有效: 事件可以在空间上彼此“相邻”，也可以在时间上彼此“相邻”。广义相对论的整体时空结构并非如此，它仅在近似下涌现。广义协变动力学形式体系已经定义清晰: 它建立在部分可观测量的概念之上——部分可观测量是可测量但一般无法单独预测的量。动力学理论给出了这些量在经典和量子领域的关联。量子引力的可观测量与广义相对论的可观测量完全一致: 原则上，相对论引力物理学中的任何测量在量子引力中同样是有效的测量。(相对论引力物理学中的任何测量都可以表示为按照上文描述的形式在三维曲面上进行的测量)。

The entire theory has a strong relational character: localization in space and time is relational. Measurements imply relations between spacetime regions. Evolution is given as relative evolution. Quantum states are interpreted as relative states in the relational interpretation of quantum theory. (The entropy gradient as well could be a perspectival phenomenon [85].) This deeply relational aspect of reality that comes both from general relativity and from quantum mechanics and that merges so naturally in quantum gravity is perhaps the deepest insight that quantum gravity is offering into the nature or reality [99].

整个理论具有鲜明的关系性特征: 时空定位是关系性的，测量意味着时空区域之间的关系，演化表现为相对演化。在量子理论的关系性诠释中，量子态被诠释为相对态。(熵梯度也可能是一种视角性现象 [85]。) 这种深深植根于广义相对论与量子力学的关系性本质，在量子引力中如此自然地融合在一起，或许就是量子引力带给我们的关于实在本性的最深刻洞见 [99]。

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